ANSWERS

Standard
MYP Mathematics
A concept-based approach

ENHANCED ONLINE

Years 4&5

Rose Harrison
Clara Huizink
Aidan Sproat-Clements
Marlene Torres-Skoumal

OXFORD

Unit 1 Answers

1.1

You should already know how to:

1 17.3 cm

2 a $P = 15.4$ cm, $A = 10$ cm^2 **b** $P = 18.8$ cm, $A = 28.3$ cm^2

3 a 120 cm^3 **b** 402 cm^3

4 $4x + 20 = 180$; $x = 40°$

Practice 1

1 240 000 000 = 2.4×10^8 times around the Earth (2 s.f.)

2 1 month and 1 day **3** 14 km/h

4 A: 80 ml; B: 120 ml **5** 23 pieces of jewellery

6 11.0 mph

Practice 2

1 48 students

2 a 92% **b** 91% **c** 4 more tests

3 20 minutes

4 8 minutes 20 seconds

5 a 4 hours 48 minutes
b 10 hours 30 minutes

6 176 steps

7 120 more days

8 a 170 diagonals **b** $\frac{n(n-3)}{2}$ diagonals

Practice 3

1 75 cm^3

2 a 1 : 6.29
b The ratio is about $1:2\pi$
c Student's own explanation.

Practice 4

1 25π cm^2

2 1000 cm^3

3 length = 3 cm, width = 12 cm, height = 18 cm

4 Its volume will decrease by $\frac{1}{16}$

5 40 minutes

6 358 cm^3

7 Use the 4L container to fill the 9L container, leaving 3L in the 4L container.
Empty the 9L container.
Pour the 3L from the 4L container into the 9L container.
Use the 4L container to fill the remaining 6L of the 9L container, leaving 2L in the 4L container.
Empty the 9L container.
Pour the 2L from the 4L container into the 9L container.
Fill the 4L container and pour it into the 9L container.
The 9L container will now contain the 6L you need.

Practice 5

1 You need the base and the perpendicular height.

2 The $4 per ticket is irrelevant. You need to know how much she paid.

3 You need to know the depth, length and width of the TV.

4 The time he gets up is irrelevant. You need to know his walking speed.

5 The bottle's mass is irrelevant.

6 The take-off time and speed are irrelevant. You need no more information, ignoring the curvature of the Earth.

Mixed practice

1 2 **2** $x = \frac{360°}{7} = 51.4°$

3 $\frac{7}{13}$ **4** 4 hours

5 3 hours **6** 4 mg

7 47.5% **8** 12 hours

9 $\frac{5}{6}$ of the catalogue **10 a** after 5 months **b** 330 hamsters

11 36 apples **12** 510 000 Euros

Review in context

1 a i 128 grains of rice
　　ii 32 768 grains of rice
　　iii 9.22×10^{18} grains of rice
　b i 2.56 cm^2
　　ii 655.36 cm^2
　　iii 167 772.16 cm^2
　c 16.78 m^2
　d 18 446 744.07 km^2
　e i 255 grains of rice
　　ii 65 535 grains of rice
　　iii 16 777 215 grains of rice
　　iv 1.845×10^{19} grains of rice

2 a i 7 moves
　　ii 15 moves
　　iii 31 moves
　b i 7 seconds
　　ii 31 seconds
　　iii 63 seconds = 1 minute 3 seconds
　c i 1023 seconds = 17 min 3 seconds
　　ii 1 048 575 seconds = 12 days 3 h 16 min 15 s

3 a Student's own answer.
　b $2\pi r$
　c $2\pi(r + 1 \text{ inch}) = 2\pi r + 2\pi$ inches
　d 2π inches
　e No, the size of the sphere does not matter.
　　The circumference increases by 2π inches independently of the radius r.

There may be more solutions to question 4 than provided here.

4 a $(7 + 1) \times (2 + 1) = 24$ **b** $(2 + 2 + 4) \times 3 = 24$
　c $(2 + 3 \times 6) + 4 = 24$ or $(3 - 2) \times 4 \times 6 = 24$
　d $(12 + 7 \times 2) - 2 = 24$

1.2

You should already know how to:

1 a Real, Rational, Integers, Natural
  b Real, Rational, Integers
  c Real, Rational, Integers
  d Real, Rational
  e Real, Irrational
  f Real, Rational
  g Real, Irrational
  h Real, Rational, Integers, Natural
  i Real, Rational
  j Real, Rational
  k Real, Rational
  l Real, Rational

Practice 1

1 a {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}; $n(A) = 7$
  b {May, June, July, August}; $n(B) = 4$
  c {1, 2, 3, 4, 6, 12}; $n(C) = 6$
  d {4, 8, 12, 16, 20, 24, 28}; $n(D) = 7$

2 a J is the set of odd positive integers less than 10.
  b K is the set of types of triangle.
  c L is the set of types of angle, not including straight line or full turn.
  d M is the set of multiples of 4 less than or equal to 40.

3 a True **b** False: $7 \in C$
  c False: $1 \notin A$ **d** False: $27 \notin A$
  e False: $8 \notin D$ **f** False: $n(C) \neq n(D)$

Practice 2

1 a {−1, 0, 1, 2, 3, 4}; Finite; 6
  b {1, 2, 3, …}; Infinite
  c {5, 10, 15, …}; Infinite
  d {1, 2, 4, 7, 14, 28}; Finite; 6
  e {1, 2, 3, 4}; Finite; 4
  f {Red, Blue, Yellow}; Finite; 3

2 a $\{s \mid s = x^2, x \in \mathbb{N} \text{ and } x \text{ is odd}\}$ **b** $\{t \mid t = 10x, x \in \mathbb{Z}\}$
  c $\{u \mid u \in \mathbb{R}, 1 < u \leq 2\}$ **d** $\{v \mid v \in \mathbb{Q}, 0 < v < 1\}$

3 For this question, answers other than those below are possible.
  a $\{x \mid x = 2n - 1, x \in \mathbb{N}\}$ **b** $\{x \mid x = 3n, x \in \mathbb{N}\}$
  c $\{x \mid x = 2^n, n \in \mathbb{Z}, 0 \leq v \leq 5\}$

Practice 3

1 a Yes **b** No **c** No

2 a True **b** True
  c False: 3 should not be bracketed
  d False: 4 should be bracketed **e** True
  f True **g** False: 4 cannot be in both sets

3 a True
  b False: answers will vary, e.g. $R = \{1, 2\}$, $S = \{1, 2, 3\}$
  c True **d** True **e** False

4 {1, 2, 3, 4, 5}

5 24 is not a power of 2

Practice 4

1 Student's own answers

2 In subtraction, the order matters: $a - b = b - a$ is only true if $a = b$ but not for all values of a and b, and $a - (b - c) = (a - b) - c$ is only true if $c = 0$, but not for all values of a, b, and c.

3 In division, the order matters: $a \div b = b \div a$ is only true if $a = b$ and a, $b \neq 0$, and $a \div (b \div c) = (a \div b) \div c$ is only true if $c = 1$ and b, $c \neq 0$, but not for all values of a, b, and c.

4 a Commutative property of addition
  b Commutative property of multiplication
  c Identity under addition
  d Distributive property
  e Identity under multiplication
  f Associative property of addition
  g Commutative property of addition
  h Associative property of multiplication

5 a Distributive property
  b Distributive property (twice)
  c Associative property of addition
  d Sum of $5x + 4x$ is $9x$ (adding like terms)

Practice 5

1 a 40 000 **b** 690 000 **c** 10 000 000 **d** 60 000

2 a 3000 **b** 14 100 **c** 900 **d** 6800

3 a 65 000 **b** 689 000 **c** 10 000 000 **d** 64 000

4 a 12.1 **b** 324.6 **c** 23.6 **d** 34.7 **e** 1.4

5 a 12.08 **b** 324.57 **c** 23.61 **d** 34.66 **e** 1.40

6 a 23 900 **b** 34 600 **c** 421 000 **d** 2310
  e 26 700 **f** 0.0346 **g** 0.000324

7 a 24 000 **b** 35 000 **c** 420 000 **d** 2300
  e 27 000 **f** 0.035 **g** 0.00032

8 a

	2 decimal places	3 decimal places	3 significant figures
3.141 592 5	3.14	3.142	3.14
2.718 28	2.72	2.718	2.72
234.5678	234.57	234.568	235
3.1782	3.18	3.178	3.18
51.235	51.24	51.235	51.2
0.001 235 6	0.00	0.001	0.001 24
0.040 022 3	0.04	0.040	0.0400

9 a 8.05 **b** 0.908 **c** 6.45 **d** 0.717
  e 2.18 **f** 2.13

Practice 6

1 Answers may vary slightly.
  a 1000 **b** $\frac{50}{3}$ or 17 **c** 1.4
  d 240 **e** $\frac{23}{2}$ or 10 **f** 10

2 a 15 mm^2 **b** 15 cm^2 **c** 27 m^2 **d** 5 (5.42 to 2 s.f.)

3 Yes, you can afford all the items (the exact total is $46.05).

Mixed review

1 a $n(\text{D})=52$, $n(\text{R})=26$, $n(\text{B})=26$, $n(\text{H})=13$, $n(\text{Q})=4$, $n(\text{N})=40$ (or 36 if the ace doesn't have a 1 on it).

b i True – the queen of hearts is a red card
  ii False – the queen does not have a number and so is not an element of N
  iii True – the kind of hearts is a heart
  iv True – the 10 of hearts is not a queen
  v False – not all red cards are queens
  vi False – not all queens are red cards
  vii True – all hearts are red cards
  viii False – there are cards that are neither a number card nor a queen, such as a king or a jack
  ix True – a card is either black or red, but not both, and nothing else

2 a True, $1.25 = \frac{5}{4}$ which is a rational number
b False, π is an irrational number
c True, all prime numbers are natural numbers
d False, $\frac{17}{10}$ is not a natural number
e False, prime numbers are all positive integers by definition
f True, $\sqrt{196} = 14 \in \mathbb{N}$ and $\mathbb{N} \subseteq Q$

3

	1 decimal place	3 decimal places	3 significant figures
0.1825	0.2	0.183	0.183
4.253 83	4.3	4.254	4.25
15.3682	15.4	15.368	15.4
0.003 823 6	0.0	0.004	0.003 82
236 468	236 468	236 468	236 000
0.076 027	0.1	0.076	0.0760 (the trailing zero must be written)

4 a Between 5 and 6
b

n	n^2
5	25.00
5.1	26.01
5.2	27.04
5.3	28.09
5.4	29.16
5.5	30.25
5.6	31.36
5.7	32.49
5.8	33.64
5.9	34.81
6	36.00

c 5.7 because $5.7^2 = 32.49$ is closer to 32 than $5.6^2 = 31.36$
d 5.6569 – it rounds to 5.7 so the previous choice was a good one.
e 5.1 because $5.1^2 = 26.01$ is very close to 26
f 5.0990 – it rounds to 5.1 so the previous choice was a good one.

1.3

You should already know how to:

1 a 125   **b** -32 **c** 49
2 a 4^{11} **b** $(-6)^5$ **c** 12^{13} **d** $(-12)^7$ **e** 2^{12}
3 a 37 390 000   **b** 0.0092102
4 a 7.35×10^{12} **b** 3×10^{-1} **c** 7.56×10^3
5 a 1.199×10^{-4} **b** 1.0006×10^4 **c** 7.357756×10^2

Practice 1

1 a $\frac{125}{27}$ **b** 128   **c** $\frac{1}{128}$ **d** 9
e $\frac{9}{4}$ **f** $-\frac{1}{81}$ **g** 16   **h** $\frac{2}{27}$

2 Simplify the following expressions. Write down the answers with only positive exponents.
a $27a^9b^6$ **b** -1 **c** $\frac{f^7 g^4}{e^{14}}$ **d** $\frac{5h^2}{i^3}$
e $\frac{j^2}{64k^6l^2}$ **f** $\frac{-6n^2}{m^9 o^3}$ **g** $27p^3q^3$ **h** $\frac{u^6}{s^2 t^8}$

Practice 2

1 a 1.98×10^{33}g
b 3.16×10^7s
c 63241.1 or 6.32×10^4 astronomical units in a light year
d 3.2767 or 3.28 light years in a parsec
e 207 222 or 2.07×10^5 astronomical units in a parsec

2 a Pluto is 89.3 times farther away from the Sun than Mercury.
b Earth is 2.29 times farther away from the Sun than Mercury.
c Pluto is 38.9 times farther away from the Sun than Earth.

Mixed Practice

1 a 2000   **b** $\frac{9}{49}$ **c** $\frac{3}{2}$ **d** 1024   **e** 60   **f** $-\frac{9}{8}$
2 a $\frac{32x^{25}}{y^{10}}$ **b** $-\frac{y^4}{x^8}$ **c** x^6yz^5 **d** $9xy^5$
e $\frac{1}{3x^2 y^4}$ **f** $-30x^{15}y^3z$

3 $a = 3$, $b = 4$, $c = 2$, $d = 2$, $e = 0$

4 a 2150 km
b 860 km
c 4300 km
d 0.5 m/s to 120 m/s
e 1.8 km/h to 432 km/h

5 a 1.9926×10^{-23}g
b 1.6735×10^{-24}g
c 2.9913×10^{-23}g

1.4

You should already know how to:

1 Length: 34 mm, 9 km, 13 miles
Mass: 2 kg, 15 g
Volume: 5 ml, 6 liters, 8.4 cm³
24 s is the odd one out.

2 $10^3 = 1000$
$10^6 = 1\,000\,000$
$10^{-2} = 0.01$

3 0.28, 345

4 Area = 144 cm^2

5 Surface area = 94 m^2
Volume = 60 m^3

Practice 1

1 a mm   **b** mm   **c** cm
   d ml   **e** m   **f** m^2
   g m

2 a tonnes   **b** m   **c** m^2
   d km/h   **e** m   **f** m
   g m/s

Practice 2

1 a 3 500 000 tonnes   **b** 8 000 000 000 bytes
   c 0.257 meters   **d** 0.65 liters

2 a 1 000 000 000   **b** 1 000 000
   c 1000   **d** 100

3 a 2 000 000 micrometers   **b** 3200 mg
   c 2 500 000 000 microwatts
   d 500 000 000 nanoseconds

4 a 2.85 m   **b** 0.9235 g   **c** 4.358 km
   d 4 358 000 mm   **e** 726 300 cg   **f** 1245 cl
   g 18.655 l   **h** 56 cm   **i** 5000 cm
   j 0.380506 kg

5 20   **6** 1 875 000 000

Practice 3

1 a 0.0254 m   **b** 0.0353 ounces
   c 0.454 kg (3 s.f.)   **d** 0.220 UK gallons (3 s.f.)
   e 0.568 liters (3 s.f.)   **f** 0.0328 feet

2 a 80 km   **b** 31.25 miles   **c** 154 pounds
   d 15.24 cm   **e** 91.44 cm   **f** 2.84 liters
   g 16 pints   **h** 160 900 cm

3 a 58 miles   **b** 60 lbs   **c** 1 pint
   d 4 oz   **e** 20 cm   **f** 6 feet

4 a 45.72 cm, not effective   **b** feet

5 a 20 089 liters should have been added to the 7682 liters
     already in the tank.
   b 4917 liters
   c 15 172 liters

Practice 4

1 a 2000 mm^2 **b** 6 cm^2 **c** 0.5 m^2
   d 45 000 cm^2 **e** 290 mm^2 **f** 700 000 mm^2

2 a 40 000 000 cm^3 **b** 5 cm^3 **c** 2.4 m^3
   d 3 ml   **e** 2500 cm^3 **f** 10 000 liters

3 0.0005 m^3

4 a 1 dm^3 = 1000 cm^3 **b** 1000 dm^3 **c** 6000 dm^3

5 1 135 200 liters   **6** Student's own answers

Practice 5

1 196 m^2 (3 s.f.)

2 8030 m^2, 0.00803 km^2, m^2 is easier to visualize

3 a 62.4 m^2 **b** 352 m^3 (3 s.f.)

4 36 ha

Practice 6

1 a 1.25 by 1.875 miles
   b 2.34 square miles

2 a i 1966.45 cm^3 **ii** 954.84 cm^2
   b i 1966.45 cm^3 **ii** 1019.35 cm^2
   The boxes have the same volume but different surface areas.

3 81.94 cm^3

4 8.36 m^2

5 882.87 cubic feet

6 a 4 inches   **b** 10.16 cm

7 40.0 meters

Practice 7

1 a 10 000 m/h   **b** 54 000 m/h
   c 4800 m/h   **d** 280 000 m/h

2 a 1 m/s   **b** 12 m/s
   c 2.5 m/s   **d** 0.6 m/s

3 18.06 m/s   **4** 7.2 km/h

5 2700 kg/m^3 **6** 14.3 mg/cl

7 11 340 kg   **8** 9660 kg

9 No, 30 mph = 48 km/h, so the speed limit is a bit
   lower in the US than in Europe.

10 The cat is faster (48 km/h = 13.3 m/s)

11 He is not correct. It is 1.716 grams.

12 He is not correct. It will take 1 hour 13 minutes.

13 No. It travelled at just less than 340 m/s (339.13 m/s).

Mixed practice

For question 1, other answers may be acceptable.

1 a m^3 or liters   **b** km/h or mph
   c cm^3 or liters   **d** kg or pounds
   e grams or ounces   **f** km or miles

2 a 300 cm   **b** 4800 g
   c 7.6 m   **d** 9.845 km
   e 1000 cm   **f** 0.4006 kg

3 a 113.4 g   **b** 38.1 cm
   c 26.3 feet (3 s.f.)   **d** 1.32 gallons
   e 1.25 UK gallons or 1.50 US gallons   **f** 1.76 pounds

4 a 5000 mm^2 **b** 0.95 m^2 **c** 500 000 mm^2
   d 6.4 cm^3 **e** 10 ml   **f** 100 000 l

5 a 0.95 km/h   **b** 36 000 m/h   **c** 36 000 m/h
   d 2 m/s   **e** 15 m/s

6 96 doses

7 a Person A (1.88 m)   **b** Person B (Person A weighs 86.2 kg)

8 a Yes   **b** 26 Euros

9 50 373 liters

10 0.549 cubic inches

11 5.017 m^2

12 14.69 m

13 8.9 g/cm^3

14 Platinum is denser.

15 Yes, the car is exceeding the speed limit.

16 The Formula 1 racing car is faster.

Review in context

1 a i 227.38 m   **ii** 2331.72 m   **iii** 128 720 km
   b 7850 kg/m³   **c** 278 377.48 m³
   d i 53.975 mm   **ii** 32.725 mm

2 a 1435 mm
   b i 1422 mm   **ii** 13 mm   **iii** 2.4% error
   c 3 ft 3 in
   d Student's own answers, such as 'some trains travel from Spain to France.'

1.5

You should already know how to:

1 a 8   **b** 11   **c** 5   **d** 7

Practice 1

1 a 11   **b** 8   **c** 3.8–3.9
   d 3.4–3.5   **e** 5.1–5.2   **f** 7.1 (just over 7)

2 a 4.8–4.9   **b** 7.4–7.5   **c** 9.1
   d About 9.5   **e** About 7.5   **f** 9.9

3 a <   **b** <   **c** >
   d >   **e** <   **f** <

Practice 2

1 a $\sqrt{130}$ **b** $63\sqrt{105}$ **c** $\sqrt{77}$

   d 3   **e** $\frac{\sqrt{15}}{5}$ **f** $\sqrt{7}$

   g $\sqrt{21}$ **h** $\frac{1}{4}$

2 3

Practice 3

1 $2\sqrt{6}$ **2** $4\sqrt{2}$

3 $6\sqrt{2}$ **4** $5\sqrt{5}$

5 $3\sqrt{15}$ **6** $8\sqrt{15}$

7 $15\sqrt{3}$ **8** $12\sqrt{6}$

9 $\sqrt{991}$ **10** $4\sqrt{62}$

11 $3a^3\sqrt{11}$ **12** $10b^2c\sqrt{2}$

13 $12x^4y\sqrt{2y}$

Practice 4

1 $-2\sqrt{3}$ **2** $17\sqrt{17}$

3 $3\sqrt{a+3}$ **4** $6\sqrt{pq}$

5 $\sqrt{3}$ **6** $8\sqrt{3}$

7 $-5\sqrt{3}$ **8** $13\sqrt{2}+4\sqrt{3}$

9 $\sqrt{5}+\sqrt{6}$ **10** $13\sqrt{2}+7\sqrt{3}$

11 $17\sqrt{3x}$ **12** $(x-4)\sqrt{3}$

13 $\sqrt{3x}(3-x)$ **14** $-11\sqrt{x+2y}$

Mixed practice

1 >   **2** >

3 <   **4** >

5 6.7   **6** 4.1

7 7.3 or 7.4   **8** 3.4 or 3.5

9 $2\sqrt{3}$ **10** $3\sqrt{2}$

11 $5\sqrt{5}$ **12** $11x\sqrt{x}$

13 60   **14** 6

15 $x^3y^2\sqrt{y}$ **16** 7

17 $\frac{\sqrt{5x}}{x}$ **18** $\frac{\sqrt{6}}{2}$

19 $\frac{5\sqrt{3}}{3}$ **20** $10\sqrt{13}$

21 $-5\sqrt{2}$ **22** $21\sqrt{x}$

23 $16\sqrt{5t}$ **24** $16\sqrt{2a}$

1.6

You should already know how to:

1 a -2 **b** 4   **c** -15 **d** -9
2 a 11   **b** 7   **c** $\frac{1}{2}$ **d** 0.1
3 a 2   **b** 6   **c** 16
   d 4   **e** 2   **f** 8

Practice 1

1 a 35   **b** -35 **c** 234   **d** -234
   e 5.6   **f** 2.8   **g** 0   **h** $\frac{1}{2}$
   i $\frac{2}{5}$ **j** 0.01   **k** 64   **l** 6
   m -5 **n** $4\sqrt{2}$ **o** $-2\sqrt{6}$ **p** -1000

2 a 13   **b** 3   **c** 3   **d** 5
   e 2.1   **f** 18   **g** 15   **h** 6
   i 11   **j** -5 **k** 8   **l** 8

3 a 4, -4
   b Student's own answer

4 a Yes, they are driving safely, since the difference between the cars' speeds is 16 km/h.
   b Minimum = 90 km/h, Maximum = 130 km/h

Practice 2

1 a 1   **b** 15   **c** 18
   d 10   **e** 33   **f** 64

2 a -3 **b** 10   **c** 6
   d -20 **e** 6   **f** 625

3 a 2   **b** -2 **c** -1
   d 1   **e** Undefined   **f** 0

4 When $b < 5.5$

Practice 3

1 13%

2 a 240.46 ml   **b** 0.15%   **c** Yes. It's very small.

3 4.17%   **4** $23.31 or $33.54

5 $56.8 \le x \le 59.2$

6 Less than $440\,g$ or more than $560\,g$

7 500 kg measured with percentage error of 1 %; it has an absolute error of 5 kg, whereas the 30 kg measured with a 10 % error has an absolute error of 3 kg.

Mixed practice

1 a 33   **b** −2.5   **c** −5.75
   d 4.5   **e** $\frac{2}{9}$ **f** 32
   g $3\sqrt{3}$ **h** −9

2 a 20   **b** −36   **c** $-\frac{1}{2}$
   d $\frac{9}{2}$ **e** $\frac{9}{20}$ **f** 2.4
   g 70   **h** 5

3 3.15 %   **4** 4.76 %

5 a 25 Euros   **b** 10 Euros/hour
   c i 11.11 Euros
     ii 9.99 %. The true value is an hourly rate of 11.11 Euros, and her average hourly rate is 9.99 % less.
   d 28 %. The true value is an hourly rate of 7.81 Euros, and her average hourly rate is 28 % more.

Review in context

1 a 57, 3951057   **b** 0.00 %, 0.287 %
   c Percentage errors are very small because the population is extremely large. However an absolute error of 3.95 million represents quite a large number of people, even though the percentage error is less than 0.5 %.
   d 24 768 881
   e 1 351 280 062 to 1 400 817 824
   f If the government claims the official figure may be inaccurate by 1.8 % (which is 24.7 million people), then 1.4 billion is good estimate since the absolute error (24 million) is within the acceptable range, and gives an idea of the size of the total population.

g United States:
Absolute error: 5 785 956, Reliable Value: 320 million (320 million has a smaller percentage error than 1.8 %, but 300 million has a larger percentage error than 1.8 %)
Germany:
Absolute error: 1 452 394, Reliable Value: 80 million (rounded to the nearest 10 million; 80 million has a smaller percentage error than 1.8 %)
Malaysia:
Absolute error: 545 958, Reliable Value: 30 million (30 million has a smaller percentage error than 1.8 %)
Australia:
Absolute error: 431 442, Reliable Value: 24 million (24 million has a smaller percentage error than 1.8 %, but 20 million has a larger percentage error than 1.8 %)
Monaco:
Absolute error: 679, Reliable Value: 38 000 (38 000 has a smaller percentage error than 1.8 %, but 40 000 has a larger percentage error than 1.8 %)

2 a 403 597   **b** 23.9 %
   c 263 757   **d** an increase
   e Student's own answer. Ideas include the aging of the population (increase in percentage of population over 65) and the increase in the total population.

3 a 4.45 %
   b Tobias should let Felix know how the amount had been rounded and let Felix decide if it was reasonable.
   c 13 cents; 4.45 %
   d The percentage errors are the same.

Unit 2 Answers

2.1

You should already know how to:

1 a $6x - 8x^2$ **b** $4 - 11x - 3x^2$

2 a $3xy$ **b** $2x^3 - 4x$

3 a 24   **b** 18

4 a 6   **b** 5

Practice 1

1 5, 25, 125, ...
Powers of 5 always end with a 5.
When the power is > 1, the last 2 digits are always 25.

2 The digit sum of a multiple of 9 will always be a multiple of 9.

3 The difference between the sums of the entries in the two diagonals will always be one.

4 The arithmetic mean will always be greater than or equal to the geometric mean.

5 a Student's own answers
  b Student's own answers
  c There will always be a remainder of 1.

6 a Student's own answers
  b Student's own answers
  c Student's own answers
  d The product will always be equal to the LCM multiplied by the GCD.

7 a $14 = 7 + 7$, $16 = 13 + 3$, $18 = 11 + 7$, $20 = 17 + 3$
  b **i** $30 = 23 + 7$
      ii $98 = 61 + 37$
      iii $128 = 97 + 31$
      iv 2 can't be done.
  c Every even number greater than 2 can be written as the sum of exactly two primes.

Practice 2

1 Their last digits repeat 4, 0, 6, 0
and they are always a multiple of 4

2 They are always a multiple of 3.

3 They are always a multiple of 24.

Practice 3

1 -1

2 1000

3 0.001

4 1000

5 1

Mixed practice

1 e.g. It is always a multiple of 6.

2 It is always a multiple of 5.

3 a $2^2 - 1 = 3$ which is prime.
    $2^3 - 1 = 7$ which is prime.
    $2^5 - 1 = 31$ which is prime.
    $2^7 - 1 = 127$ which is prime.
  b The evidence so far suggests that $2^n - 1$ will always be a prime whenever n is prime.
  c $2^{11} - 1 = 2047 = 23 \times 89$

4 a 1000
  b 1 000 000 000
  c $\frac{5}{3}$
  d 538

5 The total is 5 times the number in the center of the cross.

2.2

Coordinate geometry

You should already know how to:

1 Trapezoid

2 10.3 cm

3 Student's own diagram

Practice 1

1 a 5   **b** 9.43   **c** 232.4

2 They all equal $\sqrt{50}$ so the area is $50\pi = 157.1$

3 $FH = GH = \sqrt{17}$ therefore triangle FGH is isosceles.

4 $AB = \sqrt{20}$; $BC = \sqrt{45}$; $AC = \sqrt{65}$
$$\left(\sqrt{65}\right)^2 = \left(\sqrt{20}\right)^2 + \left(\sqrt{45}\right)^2$$
Therefore, ABC is a right-angled triangle with the right-angle at B.

5 a $PQ = 25$, $QR = 25$, $RS = 25$, $SP = 25$
  b Robin is not correct.
  c The second premise is false; having four equal sides makes it a rhombus.

Practice 2

1 (16, 13)

2 (7.5, −10.5)

3 a
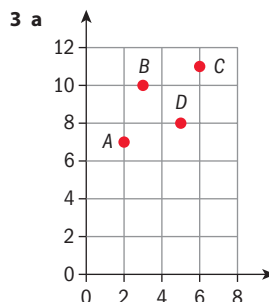
  b (4, 9), (4, 9)

c The diagonals bisect each other. All sides same length, therefore ABCD is a rhombus.

4 (5, 3)

5 $p = 3, r = 5$

6 The lengths are 26, $13\sqrt{17}$ and $13\sqrt{17}$, therefore isosceles. $h^2 = (13\sqrt{17})^2 - 13^2$ so $h = 52$ which is twice the base, hence this is also true.

Practice 3

1 a $\dfrac{-6}{5}$ **b** 2   **c** $\dfrac{5}{4}$ **d** No gradient   **e** -1 **f** 0

2 a Parallel   **b** Perpendicular
   c Parallel   **d** Perpendicular

3 $AB = \sqrt{45}; BC = \sqrt{45}; AC = \sqrt{90}$

$\left(AB\right)^2 + \left(BC\right)^2 = \left(AC\right)^2$

Therefore, ABC is a right-angled triangle with the right angle at B.

4 $b = 6$

5 $a = 1$

6 a Midpoint is the same for both diagonals, (5, 1)
   b The product of the gradients of the diagonals is -1.

7 a All sides have length 5.
   b Student's own explanation, e.g. the triangle formed by three vertices satisfies the Pythagorean Theorem.
   c Student's own explanation, e.g. the gradients of opposite sides are equal.

8 a Gradients of diagonals are $\dfrac{1}{7}$ and -7, which multiply together to make -1. Therefore the diagonals are perpendicular.
   b Midpoint of both diagonals is (-0.5, 1.5). Therefore they bisect each other.

Practice 4

1 a $y = 3x - 1$ **b** $y = -x + 2$

2 a $m = -\dfrac{1}{2}; c = 8$

   b $m = -2, c = 4$

   c $m = \dfrac{2}{3}, c = -2$

   d $m = \dfrac{1}{2}, c = \dfrac{5}{2}$

   e $m = \dfrac{1}{3}, c = \dfrac{1}{2}$

3 Student's own sketches

Practice 5

1 a $y = 2x + 4$
   b $y = -x - 1$
   c $y = x + 2$
   d $y = \dfrac{1}{6}x - \dfrac{11}{6}$

2 $y = -2x + 5$

3 $y = \dfrac{1}{2}x - 5$

4 $y = x + 1$

5 $y = 4x + 3$

6 Midpoint is (2.5, 6)
   $2x + 1 = 5 + 1 = 6 = y$

7 a The relationship is linear.
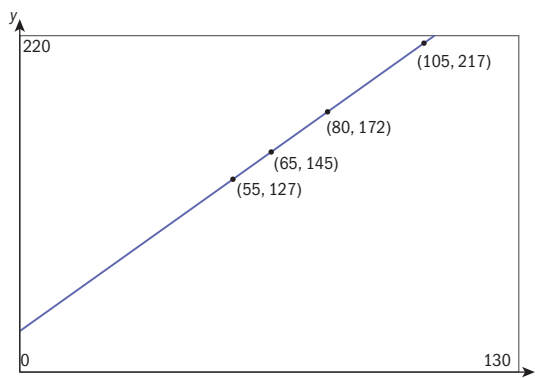
b $p = 1.8; q = 32$
   c The rate of change of degrees Fahrenheit in relation to degrees Celsius is constant at 1.8. 0 Celsius is 32 Fahrenheit.
   d 82.4 degrees Fahrenheit
   e 4.4 degrees Celsius

Mixed practice

1 a 13   **b** 5   **c** 6   **d** 25

2 a 7.81   **b** 11.4   **c** 6.45

3 a $\sqrt{80}$ **b** $\sqrt{185}$ **c** $\sqrt{117}$

4 Distance (0, 0) to (6, 0) = 6
   Distance (0, 0) to (3, 4) = 5
   Distance (3, 4) to (6, 0) = 5
   Two sides are the same length, therefore it is an isosceles triangle.

5 Distance $AB = \sqrt{50}$

   Distance $BC = \sqrt{40}$
   Distance $AC = \sqrt{50}$
   Two sides are the same length, therefore ABC is an isosceles triangle.

6 a $M = \left(\dfrac{a+c}{2}, \dfrac{b+d}{2}\right)$

   b $AB = \sqrt{(c-a)^2 + (d-b)^2}$

   c, d $AM = \dfrac{1}{2}\sqrt{(c-a)^2 + (d-b)^2}$

7 a $AC = 8, AB = 9.85 = BC$
   b Midpoints: (5, 0), (3, 4.5), (7, 4.5)
   c Distance (3, 4.5) to (7, 4.5) = 4 units
       Distance (5, 0) to (3, 4.5) = 4.92 = distance (5, 0) to (7, 4.5)
   d Triangle has base 4 and perpendicular height 4.5, area = 9

8 i 3.5   **ii** $-\dfrac{7}{3}$

9 $p = 2; q = 9$

10 $-4x + 5y - 23 = 0$

8 Answers

11 a

b The points lie on the same straight line.

c $C = 0.4n + 10.6$

d The cost of gas units increases by 40 cents for every unit of gas.

e The base cost for units of gas is 10 Euros and 60 cents.

Review in context

1 $(9, 9)$

2 $(9.5, 4)$

3 $(10, 12)$

4 The crown is at Mermaid Pool.
This is because the distance from Falling Palms to Grey Cliffs is $\sqrt{7^2 + 1^2} = \sqrt{50} \approx 7.07$ leagues.
The distance from Kaspar's cave to Mermaid Pool is $\sqrt{5^2 + 5^2} = \sqrt{50} \approx 7.07$ leagues.
Drawing a circle centered on Kaspar's Cave and passing through Mermaid Pool only gives one location, so there are no others the same distance from the cave.

5 The midpoint between Kaspar's Cave and the Grey Cliffs is $\left(\frac{7+3}{2}, \frac{7+10}{2}\right) = (5, 8.5)$.
The midpoint between (5, 8.5) and Pirates' Port is (12, 8.5).

6 a 5 leagues   **b** 5 leagues

c Since it has two sides of equal length, it is isosceles.

d Since isosceles triangles have a line of symmetry through the vertex, the point on the base that is closest to the vertex is the midpoint of the base.

e (2.5, 6.5)

2.3

You should already know how to:

1 a $4x + 12$ **b** $10x - 5$ **c** $-24 + 18x$ **d** $28x + 4x^2 - 4x^3$

2 a $3(x + 2)$ **b** $5(x - 3)$ **c** $7(2 + 5x)$ **d** $17(2 - 5x)$

3 a $x = 15$ **b** $x = 13$ **c** $x = 3$ **d** $x = 56$

4 Red: $y = -x + 1$ Blue: $y = 2x + 3$

Practice 1

1 a $x = -10$ **b** $x = \frac{10}{3}$ **c** $x = -\frac{2}{3}$ **d** $x = -1$

e $x = 36$ **f** $x = 3$ **g** $x = 1$ **h** $x = \frac{1}{2}$

2 Student's own answers

3 The left-hand side equals the right-hand side – this is an example of an identity which is true for all x.

Practice 2

1 $x = 2, y = 3$ **2** $x = 4, y = 1$ **3** $x = -2, y = -3$

4 $x = -1, y = 3$ **5** $x = \frac{87}{5}, y = -\frac{68}{5}$

6 Rupa: $x + 24 - 15x = 10$ $14 = 14x$ $x = 1$
$1 + 3y = 10$ $y = 3$
George: $50 - 15y + y = 8$ $42 = 14y$ $y = 3$
$x + 9 = 10$ $x = 1$

Practice 3

1 $x = 5, y = 2$ **2** $x = -1, y = 3$

3 $x = -2, y = 4$ **4** $x = 3, y = 1$

Practice 4

1 $x = 2, y = 4$ **2** $a = \frac{24}{5}, b = -\frac{31}{5}$

3 $x = 1, y = 2$ **4** $r = 4, s = 1$

5 $x = 5, y = -2$ **6** $x = 3, y = -1$

Practice 5

1 $x = 2, y = 3$ **2** $x = 1, y = 6$ **3** $x = 2, y = 5$

4 $x = 2, y = 5$ **5** $x = 1, y = -2$ **6** $x = 3, y = 5$

Practice 6

1 $R = 8800x$; $C = 90\,000 + 7800x$
$R = C$ when $x = 90$ weeks.
90 weeks

2 $x = -\frac{1580}{1223}$ (-1.29 to 2 d.p.) so working backwards makes it part way through 2003

3 A: $y = 31.45 + 0.155x$
B: $y = 5.20 + 0.37x$
Graphing both to see where they are equal, cost of both plans is \$50.37 when $x = 122$ minutes. For $x < 122$, plan B is cheaper. For $x > 122$, plan A is cheaper.

4 \$500 in the 2% one, \$4000 in the 5% one

5 6 hours 40 minutes cycling and 3 hours 20 minutes walking

Mixed practice

1 a $x = 5$ **b** $x = \frac{7}{8}$ **c** $x = -9$ **d** $x = \frac{12}{7}$

2 a $x = -1, y = -1$ **b** $x = -9, y = -4$

c $x = \frac{18}{5}, y = \frac{3}{5}$ **d** $x = \frac{3}{8}, y = -\frac{3}{4}$

e $x = 2, y = 4$

Review in context

1 $38x + 5y = 550$; $56x + 60y = 1200$
13.5 ounces of milk and 7.4 ounces of orange juice.
This is realistic.

2 $x + y = 8$; $0.12x + 0.32y = 1.6$
4.8 liters of 12% acid and 3.2 liters of 32% acid

3 $x + y = 20$; $10.50x + 8.25y = 9(x + y)$
$6\frac{2}{3}$ kg of premium and $13\frac{1}{3}$ kg of standard

4 Laser: $y = 150 + 0.015x$
Inkjet: $y = 30 + 0.06x$
For $x \leq 2666$, the inkjet printer is cheaper.
For $x \geq 2667$, the laser printer is cheaper.

Unit 3 Answers

3.1

You should already know how to:

1 a 729 **b** 0.015625

2 a $\frac{1}{18}$ or $0.0\dot{5}$ **b** 18

3 a $\{x \mid x \in \mathbb{R}\}$ **b** $\{x \mid x \in \mathbb{Z}, x > 0\}$

Practice 1

1 a many-to-one
  b one-to-one (unless there are siblings in your class)
  c one-to-many

2 a Student's own definitions
  b Student's own relation

3 For $y = x + 2$, one-to-one
   For $x^2 = y^2$, many-to-many

Practice 2

1 a Yes. Every input value has only one output value.
  b Domain = $\{1, 2, 3, 4, 5, 6\}$, Range = $\{1, 2, 3\}$
  c i 1 **ii** 2 **iii** 2

2 a i The country where a brand of chocolate comes from.
    ii Because each brand of chocolate comes from only one country.
    iii Domain = {Neuhaus, Cote d'Or, Leonidas, Toblerone, Lindt, Hershey's, Baci}
       Range = {Belgium, Switzerland, USA, Italy}
    iv Student's own choice.
  b i y is the square of x.
    ii Because every input value has only one output value.
    iii Domain = $\{\pm 1, \pm 2, \pm 5\}$
       Range = $\{1, 4, 25\}$
    iv Student's own choice.
  c i Whether a person is a girl or a boy.
    ii Because no person can be both male and female.
    iii Domain = {Joanne, James, Jessica, Jennifer, Joseph}
       Range = {Girl, Boy}
    iv Student's own choice.

Practice 3

1 a Domain = $\{2, 4, 5, 7, 8\}$, Range = $\{1, 2, 4, 6, 9\}$
  b Domain = $\{3, 4, 6, 8\}$, Range = $\{-4, 2, 4, 7, 8\}$
  c Domain = $\{-7, -3, -2, 3, 5, 8\}$, Range = $\{-1, 1\}$
  d Domain = $\{a, j, k, p, s, t\}$, Range = $\{c, g, k, p, w\}$
  e Domain = $\{a, b\}$, Range = $\{x, y, z\}$

2 a i $\{(-2, -2)\,(-1, -1)\,(0, 0)\,(1, 1)\,(2, 2)\}$
    ii $B = \{-2, -1, 0, 1, 2\}$
    iii

A	B
$-2 \to$	-2
$-1 \to$	-1
$0 \to$	0
$1 \to$	1
$2 \to$	2

    iv one-to-one
    v Yes, it is a function
  b i $\{(-2, -3)\,(-1, 0)\,(0, 3)\,(1, 6)\,(2, 9)\}$
    ii $B = \{-3, 0, 3, 6, 9\}$
    iii

A	B
$-2 \to$	-3
$-1 \to$	0
$0 \to$	3
$1 \to$	6
$2 \to$	9

    iv one-to-one
    v Yes, it is a function
  c i $\{(-2, 4)\,(-1, 1)\,(0, 0)\,(1, 1)\,(2, 4)\}$
    ii $B = \{0, 1, 4\}$
    iii

A	B
-2	0
-1	1
0	4
1	
2	

    iv many-to-one
    v Yes, it is a function

Practice 4

1 The domain is $x \in \mathbb{R}$.
   The range is $y \in \mathbb{R}$.

2 The domain is $x \in \mathbb{R}$.
   The range is $y \in \mathbb{R}$.

3 The domain is $x \in \mathbb{R}$.
   The range is $y \in \mathbb{R}$.

4 The domain is $x \in \mathbb{R}$.
   The range is $\{y \mid y \in \mathbb{R}, y \geq 0\}$.

5 The domain is $x \in \mathbb{R}$.
   The range is $\{y \mid y \in \mathbb{R}, y \geq 5\}$.

6 The domain is $x \in \mathbb{R}$.
   The range is $\{y \mid y \in \mathbb{R}, y \geq -3\}$.

7 The domain is $\{x \mid x \in \mathbb{R}, x \neq 0\}$.
   The range is $\{y \mid y \in \mathbb{R}, y \neq 0\}$.

8 The domain is $\{x \mid x \in \mathbb{R}, x \neq 0\}$.
   The range is $\{y \mid y \in \mathbb{R}, y \neq 0\}$.

9 The domain is $\{x \mid x \in \mathbb{R}, x \neq -1\}$.
   The range is $\{y \mid y \in \mathbb{R}, y \neq 0\}$.

10 The domain is $\{x \mid x \in \mathbb{R}, x \neq 0\}$.
   The range is $\{y \mid y \in \mathbb{R}, y \neq 1\}$.

Practice 5

1 a Yes **b** No **c** No **d** Yes

Practice 6

1 a Yes. Each input has a single output.
  b No. −1 has two different outputs.
  c Yes. Each input has a single output.
  d No, because 9 has two different output values.
  e Yes. Each input has a single output. Also, the graph passes the vertical line test.

f No, because 2, for example, has two different output values. Also, the graph does not pass the vertical line test.

g No, because 4, for example, has two different output values. Also, the graph does not pass the vertical line test.

h Yes. Each input has a single output. Also, the graph passes the vertical line test.

Practice 7

1 a $f(x) = 30x + 40$
b Domain $x \geq 0$
Range $f(x) \geq 40$
c It is a function because each input has a unique output.

2 a $f(x) = 120 - 25x$
b Domain $0 \leq x \leq 4.8$
Range $0 \leq f(x) \leq 120$
c It is a function because each input has a unique output.

3 a $f(x) = 230x$
b Domain $x \geq 0$
Range $f(x) \geq 0$
c It is a function because each input has a unique output.

Practice 8

1 a It multiplies it by 4, then subtracts 2.
b Student's own table of values.

2 a It squares it, then adds 2.
b Student's own table of values.

3 a It divides 2 by the input value.
b Student's own table of values.

4 a 18 **b** −10 **c** −3 **d** 137

5 a quadrilateral **b** hexagon
c octagon **d** decagon

Practice 9

1 a 2 **b** −2 **c** 6
d −78 **e** $2 + 4a$ **f** $2 - 8x$

2 a −5 **b** 7 **c** −17
d 31 **e** $9x - 5$ **f** $3x - 2$

3 a 13 **b** 13 **c** 13
d 13 **e** 13 **f** 13

4 a 1 **b** 26 **c** 26
d 82 **e** $x^2 + 2x + 2$ **f** $16x^2 + 1$

5 a 0 **b** 1 **c** 1
d 36 **e** 121 **f** $9x^2 - 6x + 1$

6 a 29 **b** −1 **c** 0.5
d 103 **e** −27 **f** $-4x^3 + 2x + 1$

7 a 20 **b** 35 **c** $6x + 5$
d $9x + 5$ **e** $3x + 20$ **f** $9x - 1$

8 2 **9** 0 **10** 6

11 6 or −6 **12** 7 or −7 **13** 2

14 10 **15** 315 **16** Student's mother

Mixed practice

1 a A function, because each input has a unique output.
b A function, because each input has a unique output.
c A relation, because some inputs have more than one output.
d A relation, because the input 3 has more than one output.

e A function, because each input has a unique output.
f A relation, because some inputs have more than one output.
g A function, because each input has a unique output.
h Not a function, as e.g. $x = 0$ maps to 3 and −3. Also, the graph does not pass the vertical line test.
i Not a function, as $x = 1$ maps to all values between 0 and 1. Also, the graph does not pass the vertical line test.
j A function, as e.g. $x = 1$ maps to 5, not 10 as the open circle shows the value is not included. Also, the graph passes the vertical line test.

2 a Domain = {−2, 1, 2}, Range = {−4, 1, 4}
b Domain = {3, 4, 5}, Range = {5, 6}
c Domain = {−3, −2}, Range = {−3, −2, 5}
d Domain = {1, 2, 5}, Range = {1, 3, 8}

3 a The domain is $x \in \mathbb{R}$.
The range is $f(x) \in \mathbb{R}$.
b The domain is $\{x \mid x \in \mathbb{R}, x \geq 0\}$
The range is $\{f(x) \in \mathbb{R} \mid f(x) \geq 0\}$
c The domain is $x \in \mathbb{R}$.
The range is $f(x) \in \mathbb{R}$.
d The domain is $\{x \mid x \in \mathbb{R}, x \neq 0\}$
The range is $\{f(x) \in \mathbb{R} \mid f(x) \neq 0\}$
e The domain is $\{x \mid x \in \mathbb{R}, x \neq 4\}$
The range is $\{f(x) \in \mathbb{R} \mid f(x) \neq 0\}$
f The domain is $x \in \mathbb{R}$.
The range is $\{f(x) \in \mathbb{R} \mid f(x) \geq -1\}$

4 a i −2 **ii** 2 **iii** −2
b i 28 **ii** 20 **iii** 4
c i 0 **ii** 2
iii undefined for real numbers
d i 11 **ii** $8x + 3$ **iii** $7 - 2x$

5 a i $x = 5$ **ii** $x = 0.5$ **iii** $x = 1$
b i $x = 5$ **ii** $x = -4$ **iii** $x = 0.5$
c i $x = 25$ **ii** $x = 1$ **iii** $x = 81$

6 a Because a word only ever has one amount of letters in it.
b $p(\text{word}) = $ number of letters
c i 5 **ii** 11 **iii** 2
d i Student's own 3-letter word
ii Student's own 8-letter word
e 5 **f** 4

7 a Each value n maps to one and only one value $T(n)$.
b Domain = {−3, −2, −1, 0, 1, 2, 3, 4, 5, 6, 7, 8} (ground floor is zero, so the 9 floors above ground are 0 to 8)
c Student's suggestions for values of $T(n) \geq 0$
d Range $T(n) \geq 0$ with maximum value and suitable justification. E.g. $0 \leq T(n) < 1440$ because there are 1440 minutes in 24 hours and the lift will not return to a floor in less than one minute.

8 a, b Mass determines price
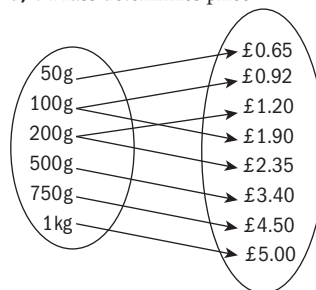
c Not a function, because e.g. 100 g maps to two possible prices.

Review in context

1 a
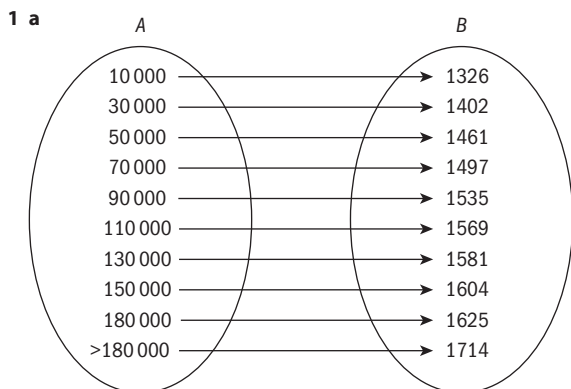
b Yes, it is a function.

c Higher income may enable the student to have access to tutoring or better books etc. or the higher income may be a result of higher intelligence.

d Student's own decision and explanation.

2 a Input = Percentage of world population,
Output = Percentage of world's resources used.
Domain = $\{x \mid x \in \mathbb{R}, 0 \le x \le 100\}$
Range = $\{y \mid y \in \mathbb{R}, 0 \le y \le 100\}$

b No, because the input value 3, for percentage of world population, has multiple output values.

c
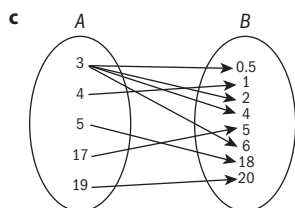
d Student's own opinion.

e Student's own opinion.

3 a $C = 24\,000 - 1000a$

b $C(18)$ represents the average cost of car insurance for an 18 year old male.
$C(18) = \$6000$

c Yes

d Student's own explanation.

e $C(21)$ represents the cost of car insurance for a 21 year old male.
$C(21) = \$3000$

f Because otherwise it would have a negative cost over the age of 24.

3.2

You should already know how to:

1 a $x^2 + 7x + 12$ **b** $x^2 - 4x - 5$
c $x^2 - 5x + 6$ **d** $x^2 - 4$

2 a $3(x + 4)$ **b** $x(x + 5)$
c $3(2x^2 + x + 4)$ **d** $(4x + 7)(3x - 5)$

Practice 1

1

p	p	a	b	c
2	−3	1	−1	−6
4	−2	1	2	−8
8	−8	1	0	−64
−3	−2	1	−5	6
−3	−7	1	−10	21
9	−9	1	0	−81
−6	4	1	−2	−24

2 $p = q = 2$ and $b = c = 4$

3 Yes, as $pq = 0$

Practice 2

1 a $(x + 3)(x + 4)$ **b** $(x + 3)(x + 5)$ **c** $(x + 6)(x - 3)$
d $(x + 9)(x - 2)$ **e** $(x - 9)(x + 2)$ **f** $(x + 7)(x - 2)$
g $(x - 4)(x - 9)$ **h** $(x - 3)(x - 8)$ **i** $x(x - 3)$

2 a $(x + 7)^2$ **b** $(x + 11)^2$ **c** $(x - 6)^2$

3 a $3(a + 1)^2$ **b** $2b(2b + 1)$
c $5(c + 3)(c - 5)$ **d** $3d(2d - 1)$
e $4(e + 9)(e - 4)$ **f** $3(f - 3)(f - 5)$

4 a $x + 5$ **b** $x + 3$

5 a $x^2 - 7x + 12 \equiv (x - 3)(x - 4)$
b $x^2 - 10x + 16 \equiv (x - 8)(x - 2)$
c $x^2 - 5x - 14 \equiv (x + 2)(x - 7)$
d $x^2 + 9x + 20 \equiv (x + 4)(x + 5)$
e $x^2 + 10x + 16 \equiv (x + 2)(x + 8)$
f $x^2 - 5x - 24 \equiv (x - 8)(x + 3)$

6 Two sides of length $(x - 7)$, and two sides of length $(x - 3)$

Practice 3

1 a $(x + 5)(x - 5)$ **b** $(x + 11)(x - 11)$
c $(2x + 3)(2x - 3)$ **d** $(x + y)(x - y)$
e $(3x + 1)(3x - 1)$ **f** $(x^2 + 1)(x + 1)(x - 1)$
g $(4x + 13)(4x - 13)$ **h** $9(3x + 1)(3x - 1)$
i $(5u + 4v)(5u - 4v)$ **j** $(4 + 7x)(4 - 7x)$
k $(4 + x^2)(2 + x)(2 - x)$ **l** $(1 + 9y^2)(1 + 3y)(1 - 3y)$

2 a $9x^2 - 100 \equiv (3x - 10)(3x + 10)$
b $25y^2 - 16 \equiv (5y - 4)(5y + 4)$
c $16a^2 - 49b^2 \equiv (4a - 7b)(4a + 7b)$
d $9u^2 - 4 \equiv (3u + 2)(3u - 2)$
e $36t^2 - 100 \equiv (3t + 5)(12t - 20)$
f $18x^2 - 32 \equiv (3x + 4)(6x - 8)$

Practice 4

1 $(2x + 3)(x + 1)$ **2** $(3x + 1)(x + 2)$

3 $(3x - 2)(x + 3)$

Practice 5

1 a 12, 20   **b** −12, 15   **c** −32, −3   **d** −125, 8

2 a $10x^2 + 31x - 14$
$= 10x^2 + 35x - 4x - 14$
$= 5x(2x + 7) - 2(2x + 7)$
$= (5x - 2)(2x + 7)$

b $4x^2 + 27x + 18$
$= 4x^2 + 3x + 24x + 18$
$= x(4x + 3) + 6(4x + 3)$
$= (x + 6)(4x + 3)$

3 a $(5x - 4)(x - 1)$ **b** $(2x + 1)(3x + 2)$
   c $(2x + 1)(2x + 3)$ **d** $(2x - 1)(3x - 4)$
   e $(4x + 3)(2x - 5)$ **f** $(3x - 2)(4x - 3)$
   g $(3x + 1)(2x + 1)$ **h** $(3x + 2)(2x - 1)$
   i $(6x + 1)(x - 1)$ **j** $(5x + 1)(3x - 2)$
   k $(4x - 1)(2x + 3)$ **l** $(7x - 3)(3x - 2)$
   m $(2x - 1)(x - 1)$ **n** $(2x - 5)(x - 2)$
   o $(3x + 1)(5x - 15)$ **p** $(x + 3)(4x - 5)$
   q $(2x - 3)(2x + 7)$ **r** $(6x + 5)(2x - 1)$
   s $(5x + 3)(x - 1)$ **t** $(7x - 2)(x - 2)$
   u $8(x + 1)(x + 2)$ **v** $(5x + 1)(x - 3)$
   w $(9x + 2)(x + 1)$ **x** $(5x + 7)(2x - 3)$

4 a $-7, -5, 5, 7$ **b** $4, 7, 9, 10$
   c Any number of the form $10n - 6n^2$ where n is a positive integer

Mixed practice

1 a $(x + 2)^2$ **b** $(x - 4)(x - 9)$
   c $(x + 7)(x - 2)$ **d** $(x + 9)^2$
   e $(x - 8)(x + 1)$ **f** $(x - 3)(x - 8)$
   g $(x - 10)^2$ **h** $(x - 20)(x + 5)$
   i $(x + 1)(x + 3)$ **j** $(x + 14)(x - 3)$
   k $(x + 12)(x - 8)$ **l** $(x + 1)(x + 6)$
   m $(x - 7)(x - 8)$ **n** $(x - 15)(x + 4)$

2 a $(x + 7)(x - 7)$ **b** $(x + 13)(x - 13)$
   c $(8 + x)(8 - x)$ **d** $(5x + 2)(5x - 2)$
   e $(12x + 9)(12x - 9)$ **f** $(16x + 13)(16x - 13)$
   g $(2x + y)(2x - y)$ **h** $(4x + 3y)(4x - 3y)$
   i $(5x + 17y)(5x - 17y)$
   j $(x^2 + 4)(x + 2)(x - 2)$
   k $(4x^2 + 1)(2x + 1)(2x - 1)$
   l $9(x^2 + 9y^2)(x + 3y)(x - 3y)$

3 a $2(x + 1)(x + 3)$ **b** $2(x + 2)(x + 5)$
   c $2(x - 1)(x - 5)$ **d** $3(x - 1)(x - 2)$
   e $2(x + 6)(x - 2)$ **f** $3(x + 8)(x - 1)$
   g $4(x + 2)(x - 4)$ **h** $2(x + 4)(x - 5)$

4 a $(2x + 1)(x + 2)$ **b** $(2x + 5)(x + 4)$
   c $(2x - 3)(x - 4)$ **d** $(3x + 1)(x - 3)$
   e $(5x - 2)(x - 2)$ **f** $(7x - 4)(x + 6)$
   g $(6x + 1)(x + 5)$ **h** $(3x - 1)(2x - 5)$
   i $(4x + 3)(2x + 3)$ **j** $(8x - 1)(x - 5)$
   k $(3x - 4)(3x + 8)$ **l** $(5x + 6)(2x - 5)$

5 a $a(4a - 3)$ **b** $7b(b + 3)(b - 8)$

6 a $(a - 9)(a + 4)$ **b** $(4b + 3)(4b - 3)$
   c $(c + 8)(c + 3)$ **d** $(4d + 3)(d - 5)$
   e $e(4e - 3)$ **f** $(f + 8)(f - 6)$
   g $(3g - 2)(g - 7)$ **h** $(4h + 1)(4h - 1)$

7 a $(n + 4)^2 - 9$
   b It factorizes to
     $(n + 4 + 3)(n + 4 - 3)$
     $= (n + 7)(n + 1)$
     which are two integers with a difference of 6.

8 a $x^2 - 8x + 15 \equiv (x - 3)(x - 5)$
   b $x^2 + 9x + 20 \equiv (x + 4)(x + 5)$

c $x^2 - 8x - 33 \equiv (x + 3)(x - 11)$
d $x^2 - 11x + 30 \equiv (x - 5)(x - 6)$

9 $(2n)^2 - 1 = (2n + 1)(2n - 1)$
which is the product of two consecutive odd integers.

10 a $2n^2 + 13n + 15$
   b Since it factorizes to give $(2n + 3)(n + 5)$, a rectangle with sides $(2n + 3)$ and $(n + 5)$ could be formed.

11 $4n^2 + 28n + 45 = (2n + 5)(2n + 9)$
The pitch is 5 m by 9 m.

12 a Sequence   8   45   112   209   336
   First difference   37   67   97   127
   Second difference   30   30   30

   2nd difference is constant.
   b $u_n = 15n^2 - 8n + 1$
   c $u_n = (5n - 1)(3n - 1)$
   d An extra 5 columns and 3 rows.

3.3

You should already know how to:

1 m is the gradient, $(0, c)$ is the y-intercept

2 a 2   **b** 4

3 a negative   **b** $y = -2x + 4$

4 a $(x + 2)(x + 3)$ **b** $(2x + 1)(x - 2)$
   c $(x - 7)(x + 7)$ **d** $(3 + x)(1 - x)$

Practice 1

1 i $(0.5, -6.25)$
   ii $x = 0.5$
   iii concave up
   iv -6
   v -2 and 3
   vi
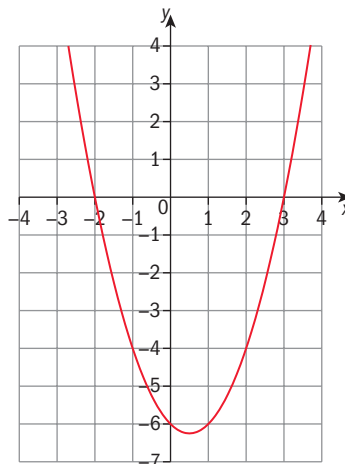
2 i $(1, -3)$
   ii $x = 1$
   iii concave down
   iv -4
   v not factorizable

vi
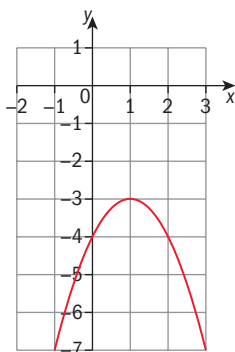
3 i $(2, -6)$
  ii $x = 2$
  iii concave up
  iv -2
  v not factorizable
  vi
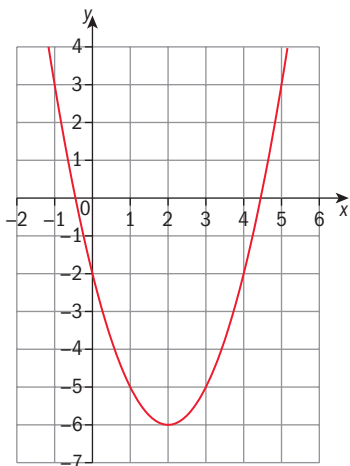
4 i $(1, -3)$
  ii $x = 1$
  iii concave down
  iv -11
  v not factorizable
  vi
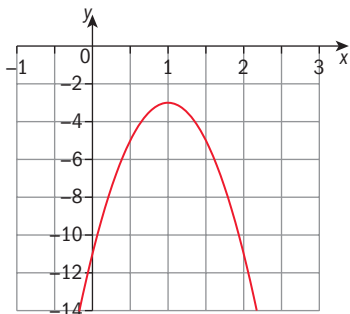
5 i $(5, -1)$
  ii $x = 5$
  iii concave down
  iv -51
  v not factorizable
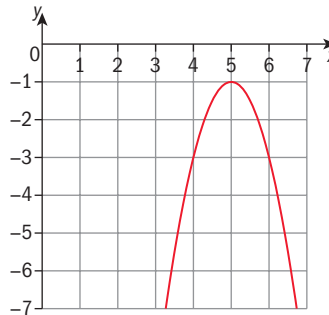
6 i $(1, -2)$
  ii $x = 1$
  iii concave up
  iv 1
  v not factorizable
  vi
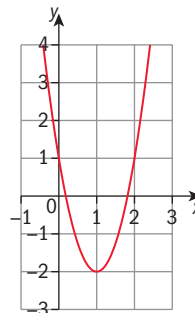
7 Student's own example, e.g. $y = x^2 - x - 6$

8 Student's own example, e.g. $y = x^2 - 8x + 16$

Practice 2

1 $T = 0.5\,°C$

2 a 45 m   **b** 1.67 sec, 58.3 m   **c** 5.15 seconds

3 44 balls

4 a 45 000 Euros   **b** 9.375 months

5 40

Practice 3

1 $(3, 1)$ concave up   **2** $(4, -3)$ concave down

3 $(-1, -1)$ concave up   **4** $(-2, 1)$ concave down

5 $(0, 2)$ concave down   **6** $(0, -1)$ concave up

7 $y = 2(x + 3)^2$

Practice 4

1 a i $y = (x + 1)^2$
    ii
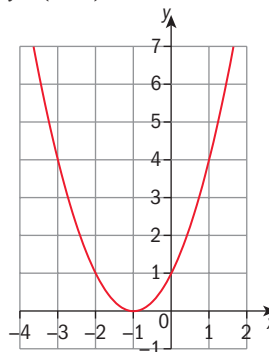
b i $y = (x-2)^2 - 6$
ii
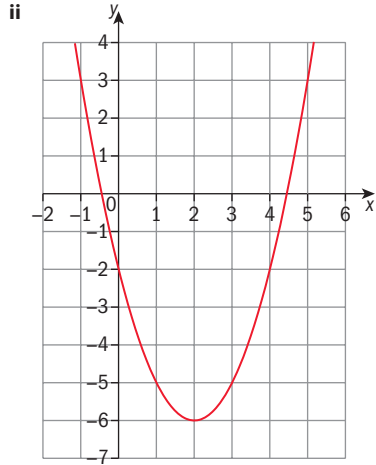
c i $y = (x+3)^2 - 6$
ii
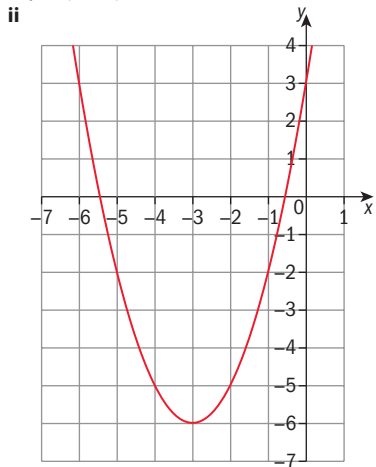
d i $y = -(x+3)^2 + 10$
ii
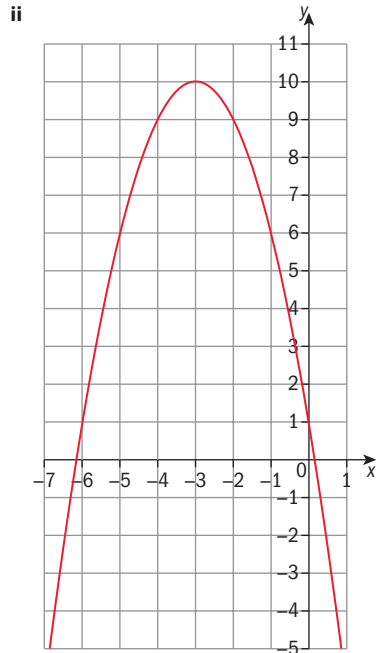
e i $y = (x-1)^2 + 2$
ii
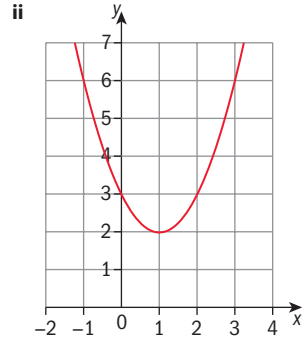
f i $y = (x-1)^2$
ii
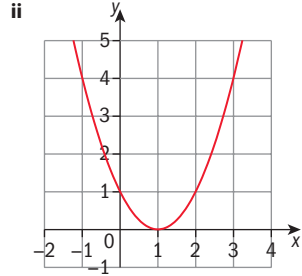
g i $y = (x+0.5)^2 + 1.75$
ii
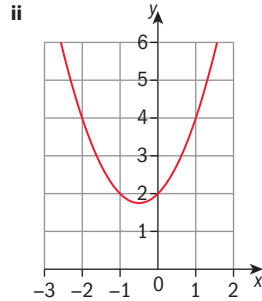
h i $y = (x-0.5)^2 - 1.25$
ii
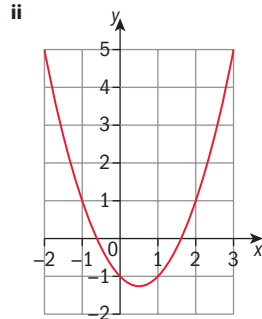
i i $y = -(x+1.5)^2 + 3.25$
ii
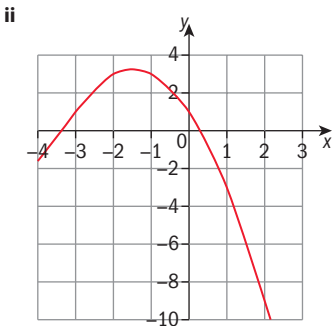
2 a i $(-2, 5)$
ii Student's own points
iii
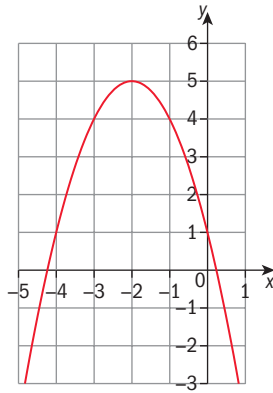
b i $(1, -1)$
ii Student's own points
iii
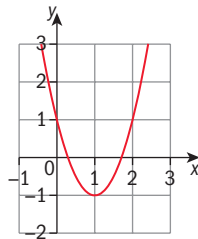
c i $(-1, 4)$
ii Student's own points
iii
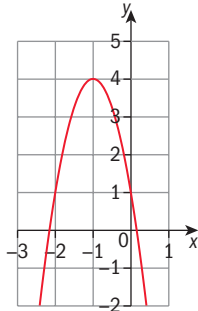
3 a C   **b** E   **c** F   **d** B   **e** A   **f** D

Practice 5

1 Maximum area = 11 250 m^2
Dimensions are 150 m by 75 m

2 Dimensions are $33\frac{1}{3}$ m by 25 m

3 Maximum area = 12.5 m^2

4 $x = 0.1$ km, $r = \frac{0.1}{\pi}$ km; Area = 9549 m^2

Practice 6

1 a–d Student's own work.

Mixed practice

1 a i vertex is $(-0.5, -12.25)$
x-intercepts are -4 and 3
y-intercept is -12

ii axis of symmetry is $x = -0.5$
It is concave up.
iii
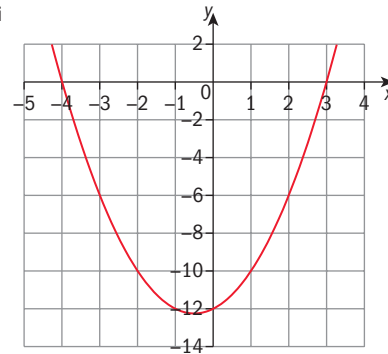
b i vertex is $(-3.5, -0.25)$
x-intercepts are -4 and -3
y-intercept is 12
ii axis of symmetry is $x = -3.5$
It is concave up.
iii
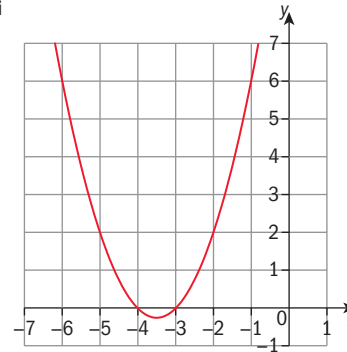
c i vertex is $(0.25, -3.125)$
x-intercepts are -1 and 1.5
y-intercept is -3
ii axis of symmetry is $x = 0.25$
It is concave up.
iii
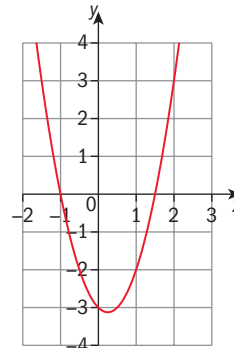
d i vertex is $\left(-\frac{1}{6}, 2\frac{1}{12}\right)$
x-intercepts are -1 and $\frac{2}{3}$
y-intercept is 2
ii axis of symmetry is $x = -\frac{1}{6}$
It is concave down.

iii
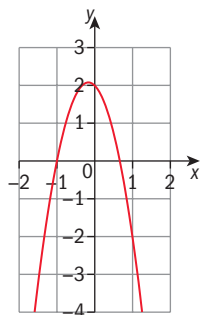
e i vertex is $\left(\dfrac{5}{12}, \dfrac{1}{24}\right)$

x-intercepts are $\dfrac{1}{3}$ and $\dfrac{1}{2}$

y-intercept is -1

ii axis of symmetry is $x = \dfrac{5}{12}$
It is concave down.

iii
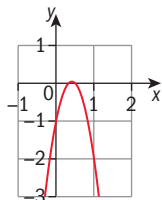
f i vertex is $(2.25, -15\frac{1}{8})$

x-intercepts are -0.5 and 5
y-intercept is -5

ii axis of symmetry is $x = 2.25$
It is concave up.

iii
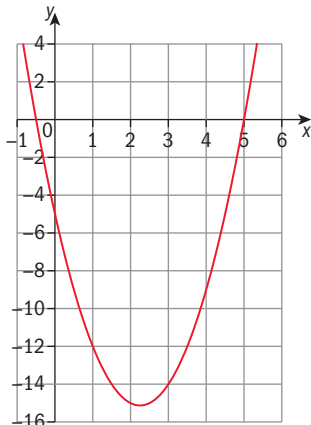
2 a $y = (x - 2)^2 + 2$
$(2, 2)$
b $y = (x + 3)^2 - 1$
$(-3, -1)$
c $y = (x + 1)^2 - 10$
$(-1, -10)$
d $y = (x - 1)^2 + 6$
$(1, 6)$
e $y = (x + 0.5)^2 - 5.25$
$(-0.5, -5.25)$
f $y = (x - 0.5)^2 + 6.75$
$(0.5, 6.75)$

3 a Area $= 300x - 3x^2$ **b** 50 m   **c** 7500 m^2

4 10 m by 10 m

5 a 176.32 months   **b** 12913
c During the 361st month

6 $y = -(x - 2)^2 + 9$ or $y = 5 + 4x - x^2$

Review in context

1 Student's own work.

2 a Answers will vary, e.g. $y = -0.408(x - 4.9)^2 + 9.8$
b A rectangle.
c Less than 9 m
d Student's own explanation.
e 1.68 m from the side of the tunnel.
f Student's own answers, e.g. dimensions of vehicles that are likely to be travelling through the tunnel.

3.4

You should already know how to:

1 a $3x^2 + 5x - 2$ **b** $x^2 - 2x - 15$

2 a $(x + 3)(x - 2)$ **b** $(2x - 1)(x + 3)$
c $(2x + 9)(2x - 9)$

3 a $(x - 3)^2 + 5$ **b** $3(x - 1)^2 + 4$ **4** $\dfrac{11}{5}$

Practice 1

1 a $0, 8$ **b** $2, 3$ **c** $-2, \dfrac{4}{3}$ **d** $0, 9$

e $\dfrac{1}{2}, -\dfrac{1}{2}$ **f** $4, -4$ **g** $-5, 2$ **h** $-\dfrac{1}{2}, \dfrac{2}{3}$

i $0, \dfrac{10}{9}$ **j** $3, -3$ **k** $-2, 2$ **l** $\dfrac{3}{2}, -6$

2 a (e.g.) $x^2 - 2x - 8 = 0$ **b** (e.g.) $2x^2 - 13x + 6 = 0$
c (e.g.) $5x^2 - 13x - 6 = 0$

3 a $x^2 + 4.25x - 3.75 = 0$ **b** $4x^2 + 17x - 15 = 0$

Practice 2

1 a -1 **b** 3 **c** 5

d $\dfrac{1}{3}$ **e** $\dfrac{3}{2}$ **f** $\dfrac{2}{3}$

2 a $x^2 + 4x + 4 = 0$
b $25x^2 - 10x + 1 = 0$
c $16x^2 + 24x + 9 = 0$

Practice 3

1 6cm, 3cm   **2** 5 cm by 9 cm

3 15 m by 28 m   **4** 22 cm, 5 cm

5 10 cm   **6** 7, 24, 25

7 8 m

Practice 4

1 $2 \pm \sqrt{3}$ **2** $-1 \pm \sqrt{2}$

3 $1 \pm \sqrt{3}$ **4** No solutions

5 No solutions   **6** $-3 \pm 2\sqrt{2}$

7 $-10 \pm 2\sqrt{15}$ **8** $\dfrac{-2 \pm \sqrt{6}}{2}$

9 $\dfrac{-3 \pm \sqrt{15}}{3}$

10 $\dfrac{-10 \pm \sqrt{94}}{2}$

11 $\dfrac{1}{2}, -\dfrac{3}{2}$

12 $\dfrac{-6 \pm 3\sqrt{2}}{2}$

Practice 5

1 a $\dfrac{-2 \pm \sqrt{5}}{2}$ **b** No solutions   **c** No solutions

   d $1 \pm \sqrt{6}$ **e** 1   **f** $\dfrac{7 \pm \sqrt{3}}{2}$

   g $\dfrac{1 \pm 2\sqrt{3}}{3}$ **h** $\dfrac{-1 \pm \sqrt{10}}{2}$ **i** $\dfrac{1 \pm \sqrt{129}}{8}$

2 a 1.3, −2.3   **b** 3.2, −1.2   **c** 0.1, −2.1
   d No solutions   **e** No solutions   **f** −1.3, 5.3

3 a $\dfrac{1 \pm \sqrt{21}}{2}$ **b** $\dfrac{-5 \pm \sqrt{29}}{2}$ **c** 1

   d $\dfrac{7 \pm \sqrt{61}}{2}$ **e** $\dfrac{3 \pm \sqrt{199}}{2}$

4 a 1.34, −1.94   **b** 14.9, 0.101   **c** 0.869, −1.54
   d 2.10, −1.27   **e** 1.66, −0.300

Practice 6

1 a i 0   **ii** No
   b i 2   **ii** Yes
   c i 2   **ii** No
   d i 0   **ii** No
   e i 2   **ii** Yes

2 a ±8   **b** ±12   **c** $k > -4$ **d** $k > \dfrac{9}{20}$

Practice 7

1 2.28 m by 4.39 m (both 3 s.f.)

2 3.49 m by 43.0 m or 21.5 m by 6.97 m (all 3 s.f.)

3 10.1 m (3 s.f.)

4 1.17 m (3 s.f.)

5 9 cm

6 8 km, 15 km

7 8.5 cm

8 a 7.5 cm   **b** 6562.5 cm²

9 8, 11 or −8, −11

10 11, 13 or −11, −13

11 12, −3

12 $2 + \sqrt{5}$ or $2 - \sqrt{5}$

Practice 8

1 Just over 14%
2 19 months
3 a

15	0	0	9500	142 500
14	1	1000	10500	147 000
13	2	2000	11500	149 500
12	3	3000	12500	150 000
p	$15 - p$	$1000(15 - p)$	$9500 + 1000(15 - p)$	$p(9500 + 1000(15 - p))$

   b $s = p(9500 + 1000(15 - p))$
   c $p(9500 + 1000(15 - p)) = 0$; $24.50
   d $12.25; 12 250
   e $9.50
   f Student's own answers

4 a $s = -3.07p + 178$ (both to 3 s.f.)
   b $R = ps$
   c $29 (nearest $)
   d $58 (nearest $)

5 a $R = (50 + 5x)(36 - 2x)$, where x is the number of price increases.
   b $70   **c** $140

6 a i 62 000   **ii** 26 000
     iii 0 (function gives −10 000)
   b $R = sp = (80\,000 - 180p)p$
   c $C = 800\,000 + 120(80\,000 - 180p)$
   d $P = (80\,000 - 180p)p - (800\,000 + 120(80\,000 - 180p))$
   e $134.33 or $430.11; $282.22

Mixed practice

1 a 6, −4   **b** 9, −3   **c** $\dfrac{4}{3}, -\dfrac{1}{2}$

   d $\dfrac{3}{2}, 7$ **e** $\dfrac{2}{5}, 1$ **f** 3, 1

2 a $-3 \pm 2\sqrt{17}$ **b** $-6 \pm \sqrt{13}$ **c** $5 \pm \sqrt{7}$

   d $\dfrac{-3 \pm \sqrt{30}}{3}$ **e** $\dfrac{4 \pm \sqrt{10}}{2}$ **f** $3, -\dfrac{1}{2}$

3 a −1.54, −8.46   **b** 6.14, −1.14   **c** 1.26, −0.26
   d 1.59, −1.26   **e** 0.12, −2.12   **f** No solutions

4 a $1, \dfrac{1}{2}$ **b** No solutions   **c** $2 \pm \sqrt{11}$

   d $\dfrac{5}{3}, -1$ **e** $3 \pm \sqrt{3}$ **f** No solutions

5 9 cm by 12 cm   **6** 5 cm by 17 cm

7 5 cm by 6 cm   **8** 3.5 m

9 12 cm and 7 cm

10 15 cm, 8 cm and 17 cm

11 9 and 11   **12** 9 or −8

13 7 and 8

14 a 5.04 seconds   **b** 0.96 seconds   **c** 3.08 seconds

15 $8.01 or $25.99   **16** 18.9 cm

Review in context

1 10.1 seconds (3 s.f.)

2 a $h = -4.9t^2 + 24t + 30$
   b i 59.4 meters (3 s.f.)
     ii 2.45 seconds (3 s.f.)
     iii 5.93 seconds (3 s.f.)

3 6 seconds

4 a 5.89 seconds (3 s.f.)
   b 6 times as long
   c Student's own answers

Unit 4 Answers

4.1
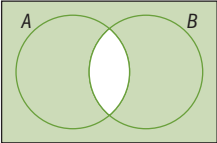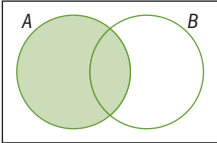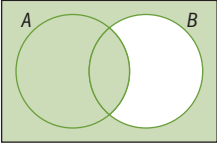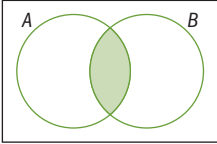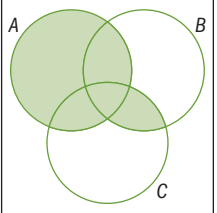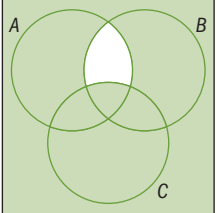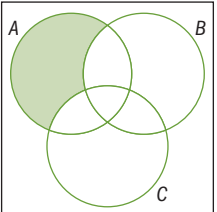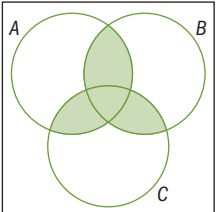
You should already know how to:

1 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

2 3, 6, 9, 12, 15, 18, 21, 24, 27, 30; 4, 8, 12, 16, 20, 24, 28

3 {Even numbers}

Practice 1
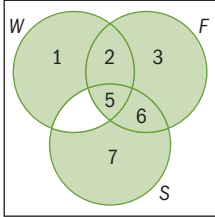
1 a $\varnothing$ **b** U **c** {3, 5, 7, 11, 13, 17, 19}
   d {1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
   e A

2 a {1, 2, 3, 4}
   b {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
   c {10, 11, 12, 13, 14, 15}
   d {5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
   e {5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

3 Answers will vary, e.g. $G = \{12, 14, 16, 18, 20\}$; $F = \{12, 14\}$; $H = \{13, 17, 19\}$

Practice 2
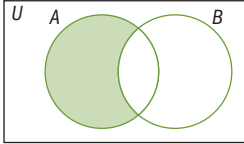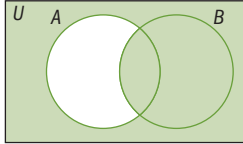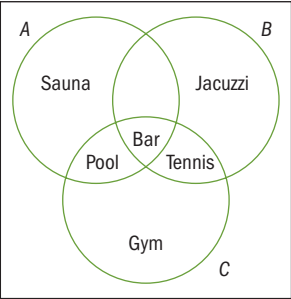
1 a $(A \cap B)'$

b $A \cup (A \cap B)$

c $(A' \cap B)'$

d $(A' \cup B')'$

e $A \cup (B \cap C)$

f $(A \cap B)' \cup C$

g $A \cap (B \cup C)'$

h $(A \cap B) \cup (A \cap C) \cup (B \cap C)$

2 a A' **b** B
   c $B \cap A'$ **d** $A \cap B \cap C$
   e $(C \cap A') \cup B$ **f** $B \cap C \cap A'$
   g $A \cup C \cup B'$ **h** $B' \cup A' \cap C'$ or $(B \cap (A \cup C))'$

Practice 3

1 a i Live birth, breathe air; lay eggs, have scales, breathe water; live in water, have fins, can swim
    ii Student's own answers

   b i

Region 1: Breathe air, live births
Region 2: Fins
Region 3: Have scales
Region 5: Live in water
Region 6: Lay eggs, breathe water
Region 7: Have legs

    ii Student's own answers

2 8

3 218

4 115

5 a 35   **b** 20   **c** 45

6 a 2   **b** 19

7 17

8 a Students who do table tennis, basketball and squash
   b Students who only do table tennis
   c Students who do table tennis and basketball but not squash

Mixed practice

1 a {7, 14}
   b {11, 13, 17, 19}
   c {even multiples of 7: 14, 28, 42, ...}
   d $\{x \mid x \in \mathbb{N}, x \neq 7\}$

2 a

b

3

4 Real Numbers

5
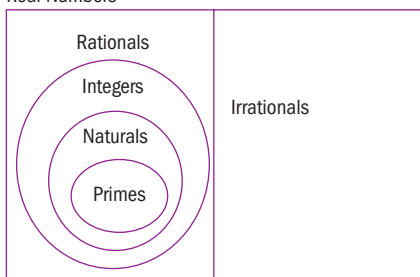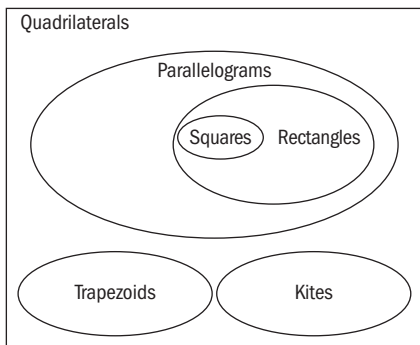
6 a {0}
  b {−10, 6, 7, 8, 9, 10}
  c {−5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5}
  d {−9, −8, −7, −6, −5, −4, −3, −2, −1}
  e {0, 1, 2, 3, 4, 5}

7 a True   **b** False: 2 ∈ {primes}
  c True   **d** True

8 13

9 a 6   **b** 3   **c** 5   **d** 21

Review in context

1 a 464, 1101, 3154   **b** 14
  c Student's own answers

2 a

A: Truths; B: Beliefs
Region 2 defines Knowledge

b
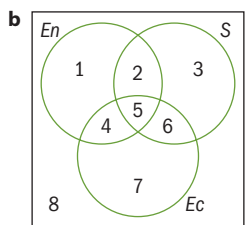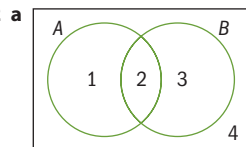
Region 5: Sustainable
Regions 5 and 2: Bearable
Regions 5 and 4: Viable
Regions 5 and 6: Equitable

3 a 10 660   **b** 18 402
  c Human and mouse: 15 213, Human and chicken: 11 705, Human and zebrafish: 12 897
  d Mouse: 82.7%, Chicken: 63.6%, Zebrafish: 70.1%
  e Mouse

4.2

You should already know how to:

1 a $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$ **d** $\frac{1}{2}$ **e** $\frac{5}{6}$

2 a $P = \{2,3,5,7\}$ **b** 1,4,6,8,9,10   **c** 4, 6

3
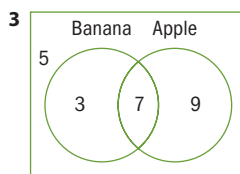
Practice 1

1 (I, I) (I, II) (I, III) (I, IV) (II, I) (II, II) (II, III) (II, IV) (III, I) (III, II) (III, III) (III, IV) (IV, I) (IV, II) (IV, III) (IV, IV)

	I	II	III	IV
I	I,I	I,II	I,III	I,IV
II	II,I	II,II	II,III	II,IV
III	III,I	III,II	III,III	III,IV
IV	IV,I	IV,II	IV,III	IV,IV

2 HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

3

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

4 (SAG, A) (SAG, O) (SAG, B) (GG, A) (GG, O) (GG, B)

5

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6

6

	1	2	3	4	5
1	1,1	1,2	1,3	1,4	1,5
2	2,1	2,2	2,3	2,4	2,5
3	3,1	3,2	3,3	3,4	3,5
4	4,1	4,2	4,3	4,4	4,5
5	5,1	5,2	5,3	5,4	5,5

7

	2	3	4	5	6
0	0	0	0	0	0
1	2	3	4	5	6
2	4	6	8	10	12
3	6	9	12	15	18
4	8	12	16	20	24

8 A tetrahedral die and a coin are both thrown.

Practice 2

1 a $\frac{1}{16}$ **b** $\frac{15}{16}$

2 a $\frac{4}{16}=\frac{1}{4}$ **b** $\frac{3}{4}$

3 a $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{8}$

Practice 3

1 a
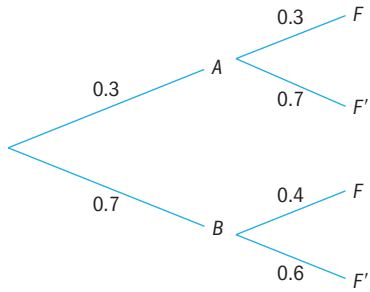
b i 0.09   **ii** 0.21

2 a
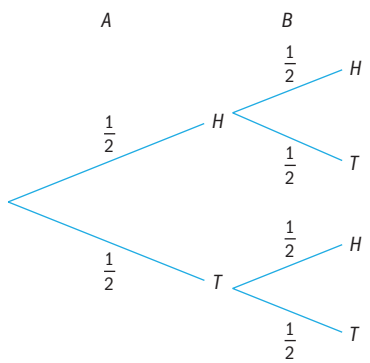
b $\frac{1}{4}$

3 $\frac{1}{36}$
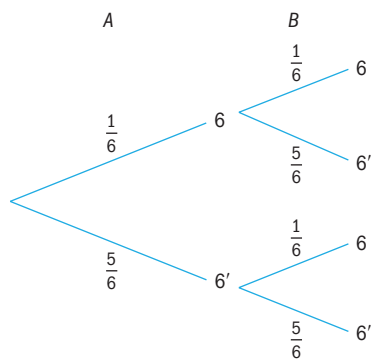
4
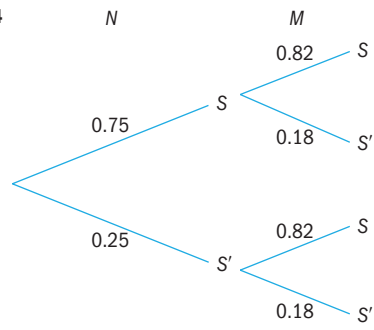
a 0.615   **b** 0.045

5 a $\frac{3}{5}, \frac{2}{5}$

b
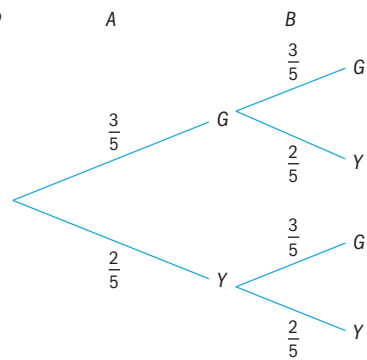
c $\frac{3}{5}\times\frac{3}{5}>\frac{2}{5}\times\frac{2}{5}$

Practice 4

1 a

b $\frac{6}{42}=\frac{1}{7}$ **c** $\frac{12}{42}=\frac{2}{7}$ **d** $\frac{30}{42}=\frac{5}{7}$

2 a

1st counter   2nd counter

$\frac{1}{7}$ W: 0 → W, $\frac{4}{6}$ → R, $\frac{2}{6}$ → B

$\frac{4}{7}$ R: $\frac{1}{6}$ → W, $\frac{3}{6}$ → R, $\frac{2}{6}$ → B

$\frac{2}{7}$ B: $\frac{1}{6}$ → W, $\frac{4}{6}$ → R, $\frac{1}{6}$ → B

b $\frac{8}{42} = \frac{4}{21}$

3

1st roll   2nd roll   3rd roll

$\frac{1}{2}$ H / $\frac{1}{2}$ T branches throughout (H → H → H/T, T; T → H → H/T, T)
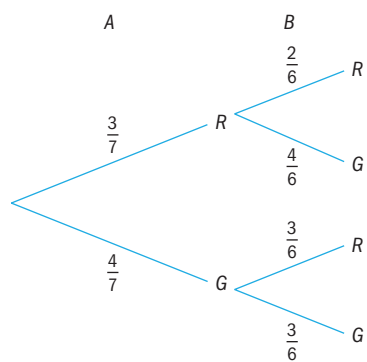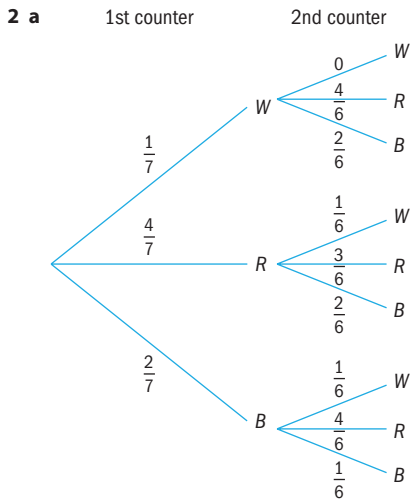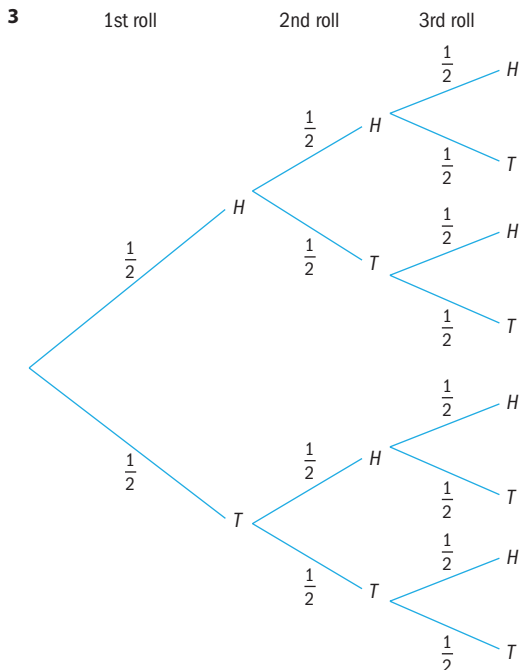
a $\frac{3}{8}$ **b** $\frac{7}{8}$ **c** $\frac{1}{8}$

d They are complementary probabilities – add to 1

4 a

A   B

$\frac{5}{9}$ O: $\frac{4}{8}$ → O, $\frac{4}{8}$ → E

$\frac{4}{9}$ E: $\frac{5}{8}$ → O, $\frac{3}{8}$ → E
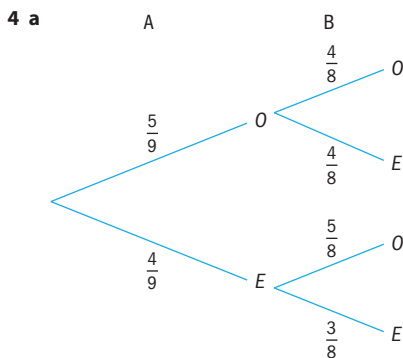
b $\frac{1}{6}$ **c** $\frac{5}{6}$

5 a 10

	J	M	R	H	P
J	–	JM	JR	JH	JP
M	MJ	–	MR	MH	MP
R	RJ	RM	–	RH	RP
H	HJ	HM	HR	–	HP
P	PJ	PM	PR	PH	–

b $\frac{1}{10}$

6 a 0.35   **b** 8

c

1st marble   2nd marble

0.4   R: 0.4 → R, 0.25 → Y, 0.35 → G

0.25   Y: 0.4 → R, 0.25 → Y, 0.35 → G

0.35   G: 0.4 → R, 0.25 → Y, 0.35 → G

d 0.64

Practice 5

1 a

Venn diagram: M and S. Outside 2; M only 28; intersection 7; S only 3.

b $\frac{28}{40} = \frac{7}{10}$

2 a

Venn diagram L, B, V: L only 5; L∩B 1; B only 9; L∩V 4; centre 2; B∩V 4; V only 2; outside 3.
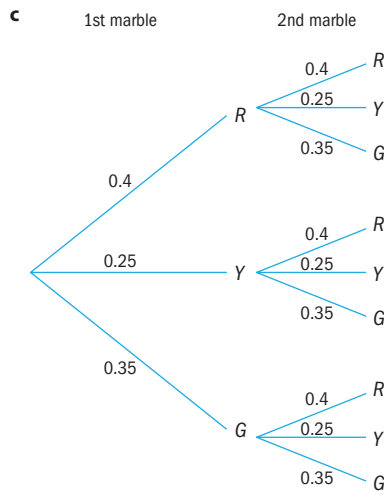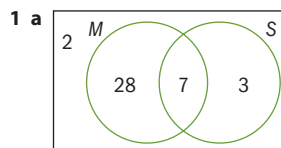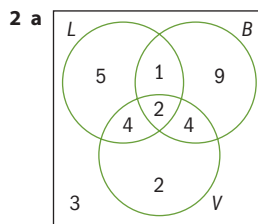
b i $\frac{4}{30} = \frac{2}{15}$ **ii** $\frac{5}{30} = \frac{1}{6}$ **iii** $\frac{2}{30} = \frac{1}{15}$

3 a

Venn diagram C, R, T: C only 6; C∩R 18; R only 6; C∩T 20; centre 26; R∩T 10; T only 4; outside 0.

b $\frac{4}{90} = \frac{2}{45}$ **c** $\frac{6}{90} = \frac{1}{15}$

4
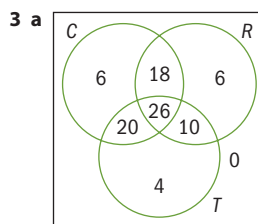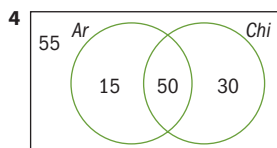
a $\frac{15}{150} = \frac{1}{10}$ **b** $\frac{30}{150} = \frac{1}{5}$ **c** $\frac{55}{150} = \frac{11}{30}$

Practice 6

1 a 29   **b** 26   **c** $\frac{14}{50} = \frac{7}{25}$

2 a

	A	I	B	Total
M	10	4	16	30
F	12	6	12	30
Total	22	10	28	60

b $\frac{10}{60} = \frac{1}{6}$

3 a 47.4% (3 s.f.)   **b** $\frac{16}{57}$ **c** $\frac{4}{57}$

4

	Under 15	15-20	Over 20	Total
M	4	6	7	17
F	8	15	11	34
Total	12	21	18	51

a i $\frac{34}{51} = \frac{2}{3}$ **ii** $\frac{6}{51} = \frac{2}{17}$ **iii** $\frac{39}{51}$

b $\frac{4}{17}$

Practice 7

1 a Yes   **b** No   **c** Yes

2 a No   **b** No   **c** No

3 a 0.2
   b No – intersection is not zero

4 a
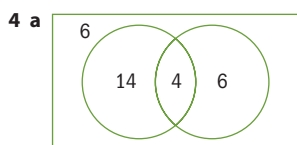
   b 4
   c Intersection is not zero, it is possible to be stung by both.

5 a $p = 4, q = 1$
   b P(boy) $= \frac{18}{30} = \frac{3}{5}$, P(girl) $= \frac{12}{30} = \frac{2}{5}$, Yes

   c P(S) $= \frac{24}{30} = \frac{4}{5}$, P($B$) $= \frac{19}{30}$, No

6 a Pick a heart, pick a diamond (for example)
   b Pick a red card, pick a 10 (for example)

Practice 8

1 Method c   **2** Student's own answers

3 a No – not enough evidence with only 10 throws for 1000 flips
   b The coin is most likely biased, as this number is too high to occur by chance.

4 a Yes it is fair

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b i No – Hadley has more chance to win
   ii In this one case, both win so 'roll again'

5 a
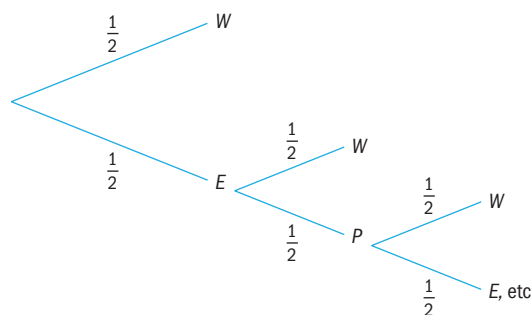
b i $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{32}$

c Not fair: Peter has a greater chance of winning than Eliott.

Mixed practice

1 a A+, A−, B+, B−, AB+, AB−, O+, O−
   b $\frac{1}{8}$ **c** $\frac{2}{8} = \frac{1}{4}$ **d** $\frac{1}{2}$ **e** $\frac{1}{2}$

2 a 12   **b** $\frac{1}{4}$ **c** $\frac{1}{6}$ **d** $\frac{1}{6}$

3 a

Ann   Simon

A tree diagram. Ann's branches: $\frac{3}{8}$ to B, $\frac{4}{8}$ to R, $\frac{1}{8}$ to G. Simon's branches from B: $\frac{2}{5}$ to B, $\frac{3}{5}$ to R. From R: $\frac{2}{5}$ to B, $\frac{3}{5}$ to R. From G: $\frac{2}{5}$ to B, $\frac{3}{5}$ to R.

b $\frac{3}{20}$ **c** $\frac{11}{20}$

4 a

Venn diagram with sets C and S: 10 outside, 2 in C only, 6 in intersection, 6 in S only.
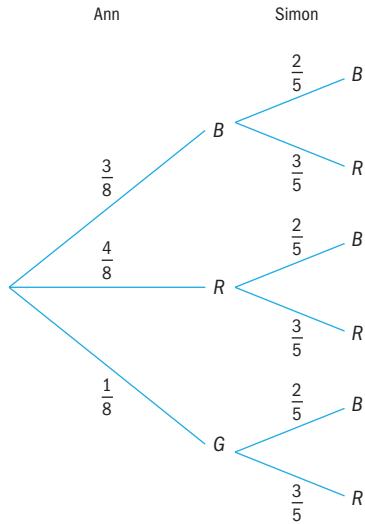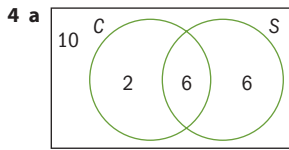
b $\frac{6}{24}=\frac{1}{4}$ **c** $\frac{2}{24}=\frac{1}{12}$ **d** $\frac{10}{24}=\frac{5}{12}$

5 a

1st item   2nd item

Tree diagram: $\frac{4}{10}$ to C, $\frac{6}{10}$ to P. From C: $\frac{3}{9}$ to C, $\frac{6}{9}$ to P. From P: $\frac{4}{9}$ to C, $\frac{5}{9}$ to P.
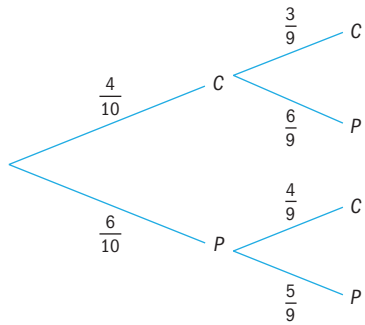
b $\frac{8}{15}$

6 a As above but second level probabilities are $\frac{4}{10},\frac{6}{10},\frac{4}{10},\frac{6}{10}$

b $\frac{12}{25}\left(\text{down by }\frac{4}{75}\right)$

7 a $\frac{23}{50}$ **b** $\frac{11}{50}$ **c** $\frac{40}{50}=\frac{4}{5}$ **d** $\frac{17}{50}$

8 a **i** $\frac{18}{30}=\frac{3}{5}$ **ii** $\frac{24}{30}=\frac{4}{5}$ **iii** $\frac{10}{30}=\frac{1}{3}$ **iv** $\frac{26}{30}=\frac{13}{15}$

b $\frac{4}{30}=\frac{2}{15}$

c

Venn diagram with sets B and F: 6 outside, 14 in B only, 4 in intersection, 6 in F only.
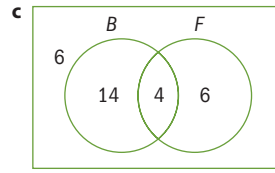
d $\frac{4}{18}=\frac{2}{9}$

9 a **i** Yes   **ii** Yes

b $\frac{108}{250}=\frac{54}{125}$ **c** $\frac{35}{106}$

d $\frac{60}{142}=\frac{30}{71}$ Not independent.

E.g. $P(\text{Male}) \times P(\text{Geneva}) \neq P(\text{Male} \cap \text{Geneva})$

10 a

Three-set Venn diagram (Av, Ma, SC): 33, 16, 10, 3, 4, 8, 7, 19.
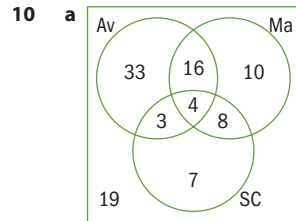
b 19   **c** No   **d** $\frac{12}{22}=\frac{6}{11}$

11 a 0.2   **b** No

12 a

Tree diagram: $\frac{1}{2}$ to G, $\frac{1}{2}$ to B. From G, six branches each $\frac{1}{6}$ to 1,2,3,4,5,6. From B, six branches each $\frac{1}{6}$ to 1,2,3,4,5,6.
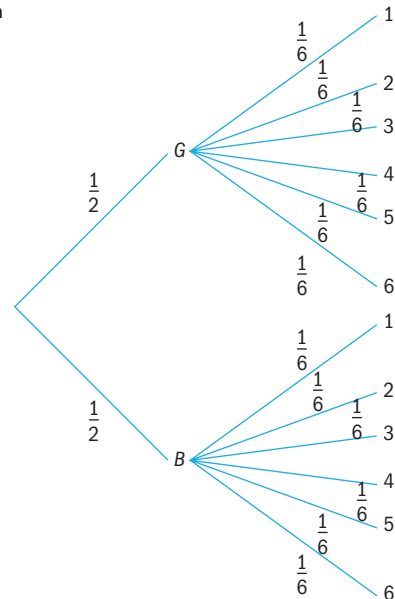
b $\frac{1}{6}$ **c** $\frac{2}{6}=\frac{1}{3}$ **d** $\frac{1}{4}$

e Yes $P(E) \times P(G) = P(E \cap G)$

13 a i $\frac{3}{50}$ **ii** $\frac{45}{50}=\frac{9}{10}$ **iii** $\frac{16}{18}=\frac{8}{9}$

b No P(M) × P(L) ≠ P(M ∩ L)

Review in context

1 a 15.8% **b** The risk increases
c About 5 times less **d** Student's own answers

2 a

		Mother					
		MM		Mm		mm	
		M	M	M	m	m	m
Father	MM	M	MM	MM	MM	Mm	Mm	Mm
		M	MM	MM	MM	Mm	Mm	Mm
	Mm	M	MM	MM	MM	Mm	Mm	Mm
		m	mM	mM	mM	mm	mm	mm
	mm	m	mM	mM	mM	mm	mm	mm
		m	mM	mM	mM	mm	mm	mm

b $\frac{9}{36}=\frac{1}{4}$ **c** $\frac{27}{36}=\frac{3}{4}$ **d**

MM	Mm
mM	mm

¼

e $\frac{6}{16}=\frac{3}{8}$ **f** $\frac{9}{16}$ **g** Mm and mm

3 a i $\frac{32}{60}=\frac{8}{15}$ **ii** $\frac{28}{60}=\frac{7}{15}$ **iii** $\frac{8}{17}$

iv $\frac{49}{60}$ **v** $\frac{3}{25}$

b i $\frac{14}{885}$ **ii** $\frac{253}{1770}$

4 a i 0.04 **ii** 0.96

b Yes **c** 0.54 **d** 0.53 **e** $\frac{42}{53}$

5 a No **b i** 0.5 **ii** 0.5

Unit 5 Answers

5.1

You should already know how to:

1 $5\sqrt{2}$ m **2** 40 cm

3 36 cm³, 84 cm² **4** 2410 m³, 1010 m²

5 56 mm³, 92 mm² **6** 61.4 cm², 15.4 cm

Practice 1

1 52 m²

2 80.9 cm²

3 65.5 cm²

4 a 7.60 m **b** 101 m²

5 84.9 cm²

6 a The triangles **b** 9140 mm²

7 20 cm

8 144 cm²

Practice 2

1 28.6 m²

2 283 cm²

3 170 cm² (There is no paper on the top circular face.)

4 a
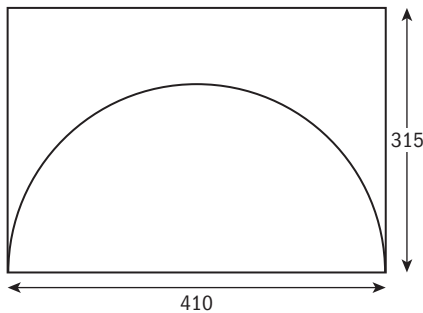
b $d = 410$ mm, $r = 205$ m **c** 66 000 mm²
d $d = 644$ mm **e** 102.5 mm

5 a 75.4 cm² **b** 6.00 cm **c** 4.48 cm

Practice 3

1 50.3 cm²

2 1810 cm²

3 a 236 cm² **b** 214 cm²

4 5.64 cm

5 The surface area of the cylinder is 1.5 times larger than the surface area of the sphere.

Practice 4

1 213 cm³ **2** 112 cm³

3 56 cm³ **4** 3620 cm³

5 314 cm³ **6** 1700 cm³

7 524 cm³ **8** 452 cm³

9 1530 cm³

10 a 1 950 000 mm³ or 1950 cm³
  b Because there is air in between the French fries, so the potato does not fill the entire cone.

11 3456 cm³

12 a They will always be enlargements of each other.
  b i 2144 cm³ **ii** 33.5 cm³

13 10 mm

14 2.00 cm (3 s.f.)

15 5.00 cm (3 s.f.)

16 The water would reach $\frac{2}{3}$ of the height of the cylinder.

Mixed practice

1 a 466 cm², 576 cm³ **b** 77.6 cm², 40 cm³
  c 118 cm², 84.8 cm³ **d** 13.1 cm², 3.14 cm³
  e 845 cm², 2310 cm³ **f** 452 cm², 905 cm³

2 a 393 cm², 407 cm³ **b** 221 cm², 172 cm³

3 a 8.00 cm (3 s.f.) **b** 163 cm³

4 Yes the soccer ball satisfies the regulations.
  It is 69.2 cm in circumference.

5 a 3.00 cm **b** 56.7 cm³

6 200 cm²

Review in context

1 a 1.58… – pretty close
  b 85 500 m²
  c Ratio is 1.61… pretty close
  d 2 570 000 m³
  e 921.6 m and 914.2 m – they are very close

2 a i 3.90 cm² **ii** 49.7 cm²
  b i 0.724 cm³ **ii** 33.0 cm³

3 Circumference of the ball is 94.3 cm so it was a pretty tight fit!

4 a The 215 cm tall tree has more area for decorations
  (21 700 cm²; the 180 cm tree has area 19 200 cm²).
  b 8100 ft² is available for decorations.

5.2

You should already know how to:

1 a, b Draw houses that are 2× and 4× larger.
2 a $y = \frac{5}{4}x - 3$
  b $x = 2$
  c $y = -3x + 3$

Practice 1

1 a $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ **b** $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

  c $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ **d** $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

  e $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$ **f** $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$

2

3

4

5

6 a kite

b

c

d

7
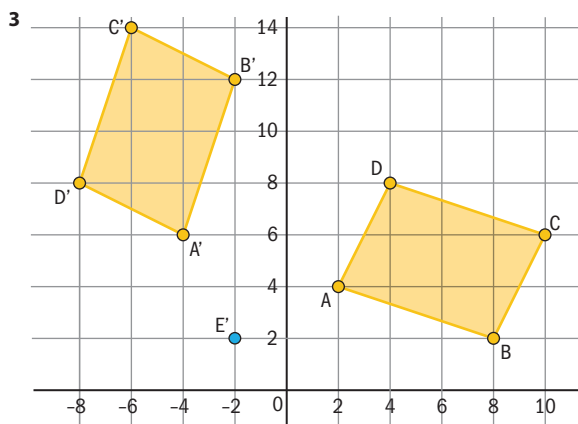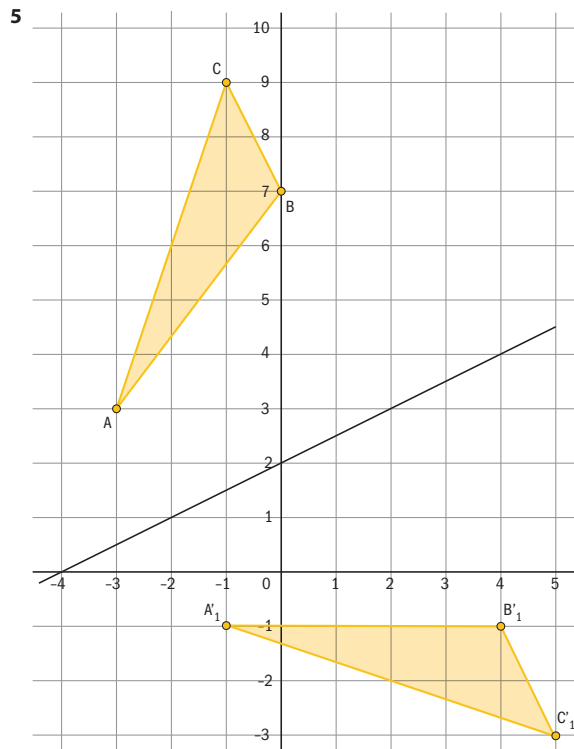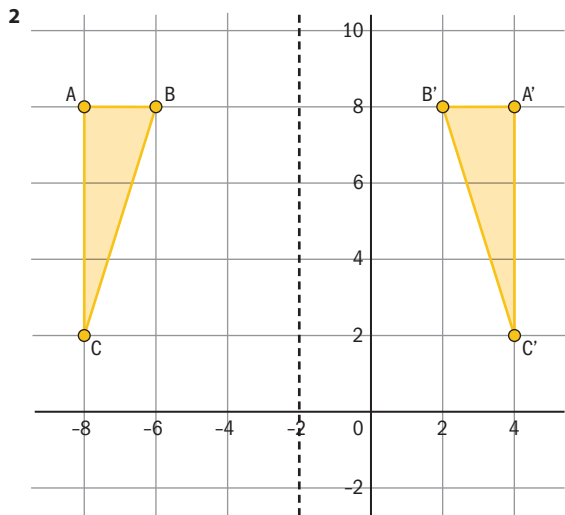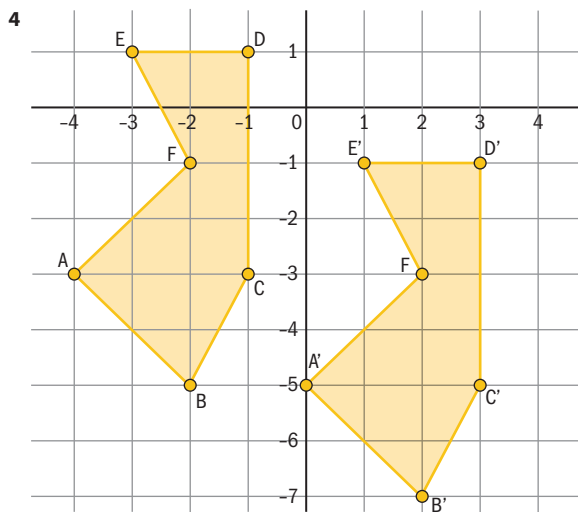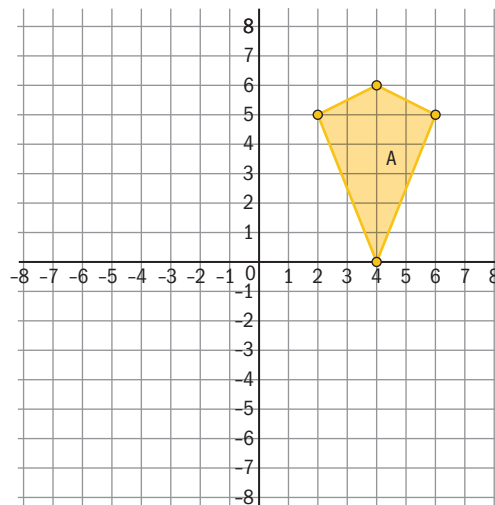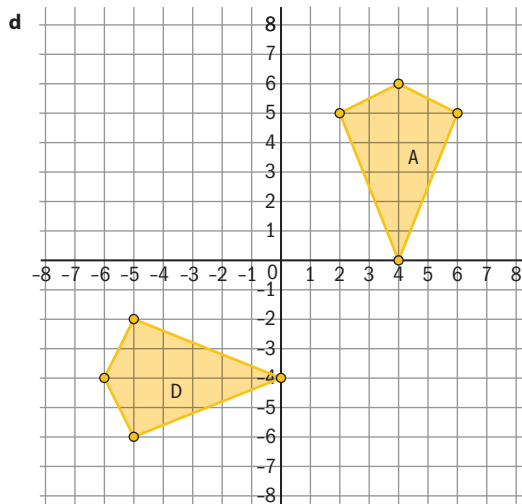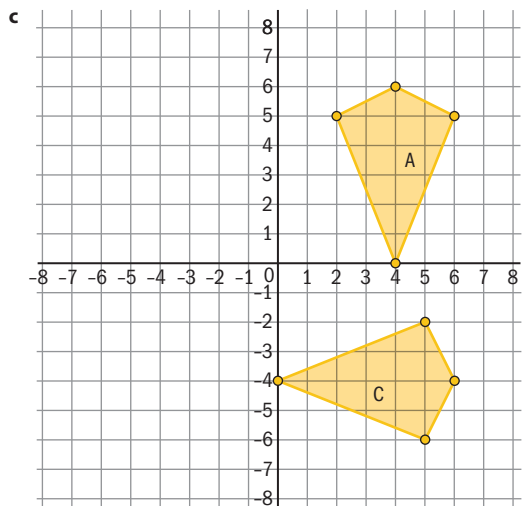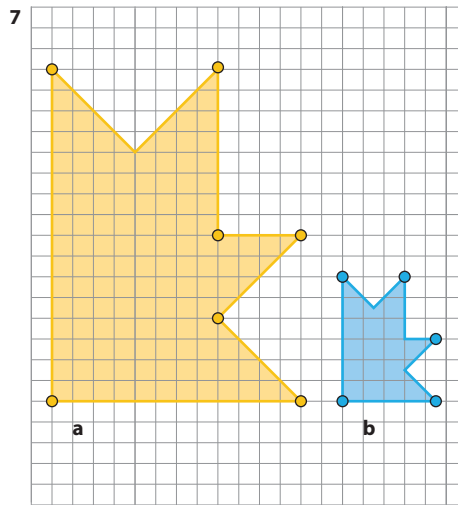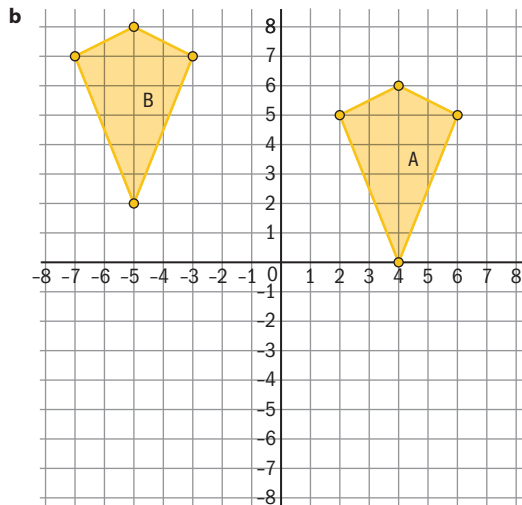
8 i Reflection: the order of the letters goes from clockwise to counterclockwise
   ii Translation: the shape maintains the same orientation
   iii Rotation by 90°: horizontal sides become vertical and vice versa
   iv Rotation by 180°: vertical sides remain vertical, but in the opposite direction (if AB goes up, then $A'B'$ goes down)

9 a $D(-1, 6)$
   b $A'(-1, -2)$, $B'(3, -2)$, $C'(3, 2)$, $D'(-1, 2)$
   c $A''(3, -2)$, $B''(-1, -2)$, $C''(-1, 2)$, $D''(3, 2)$
   d $A'''(-1, 2)$, $B'''(3, 2)$, $C'''(3, -2)$, $D'''(-1, -2)$
   e The vertices are the same 4 points for each transformation, but the order and orientation is different depending on the transformation. $A(-1, 10)$ is transformed to $A'(-1, -2)$ after the reflection, to $A''(3, -2)$ after the rotation, and to $A'''(-1, 2)$ after the translation.

10 A reflection in the line $x = 3$

11
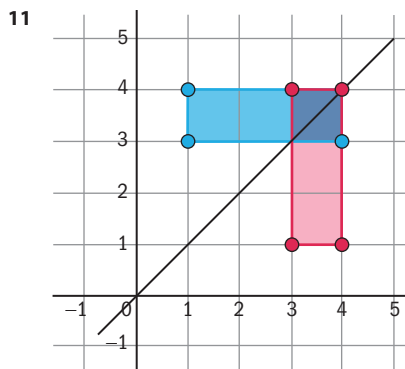
12 a Reflection in the line $y = 1$
   b Enlargement by scale factor 3
   c Rotation 90° clockwise around the point (5, 3)
   d Rotation 90° clockwise around the point (5, 5)
   e Reflection in the line $y = x - 6$

13 a 16 numbers: 00, 01, 03, 08, 10, 11, 13, 18, 30, 31, 33, 38, 80, 81, 83, 88
   b 4 numbers: 00, 11, 25, 52
   c 4 numbers: 00, 11, 69, 96
   d i 30 numbers: 01 becomes 10, 02 − 20, 05 − 50, 06 − 90, 09 − 60, 12 − 21, 15 − 51, 16 − 91, 19 − 61, 25 − 52, 26 − 92, 29 − 62, 56 − 95, 59 − 65, 66 − 99
   ii 6 numbers: 00, 11, 22, 55, 69, 96

Practice 2

1 a i Rotation by 90° counterclockwise around (0, −4)
ii Reflection in the line $y = x − 4$
iii Reflection in the line $y = −4$

b i Translation by $\begin{pmatrix} 2 \\ -14 \end{pmatrix}$

ii Rotation by 180° around (2, 0)
iii Rotation by 180° around (1, 7)

c i Translation by $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$

ii Rotation by 90° counterclockwise around (2, 5)
iii Rotation by 90° counterclockwise around (3, 0)

2 a
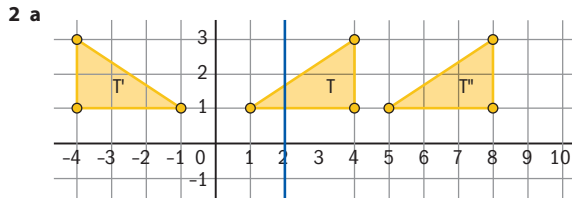
Translation by $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

b
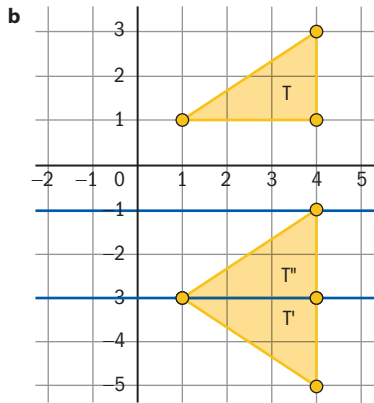
Translation by $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

c
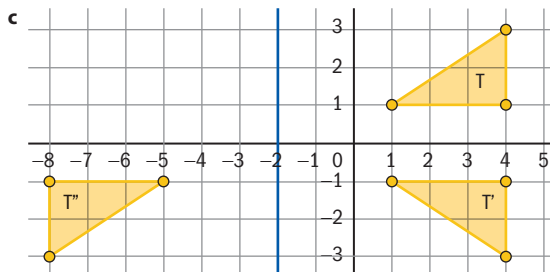
Rotation 180° around (−2, 0)

d
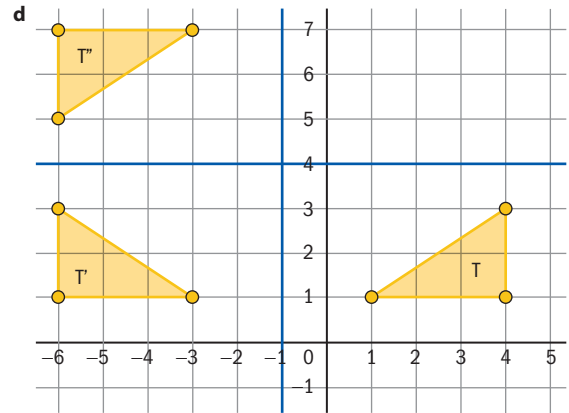
Rotation 180° around (−1, 4)

e
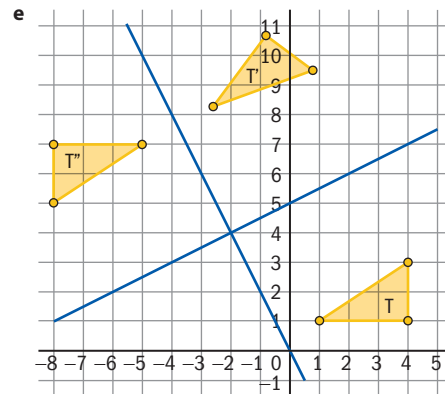
Rotation 180° around (−2, 4)

3 a
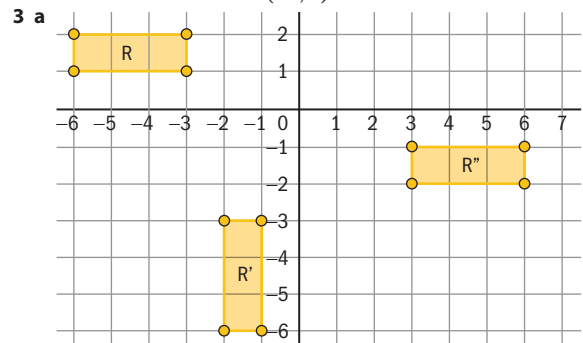
Rotation 180° around (0,0)

b
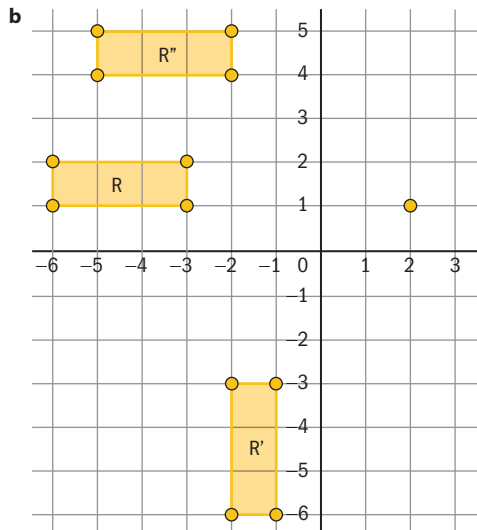
Translation by $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

c
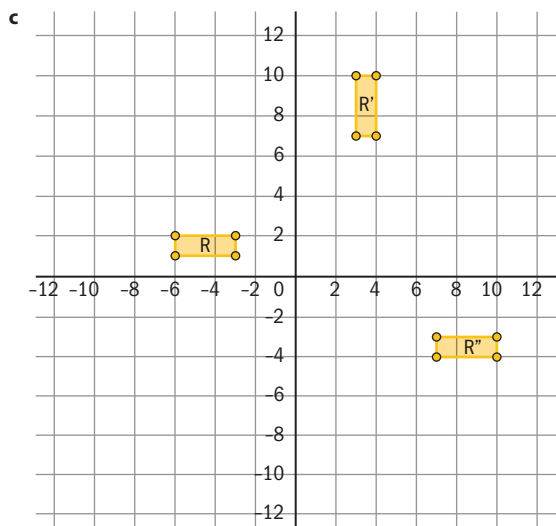
Rotation 180° around (2, −1)

d
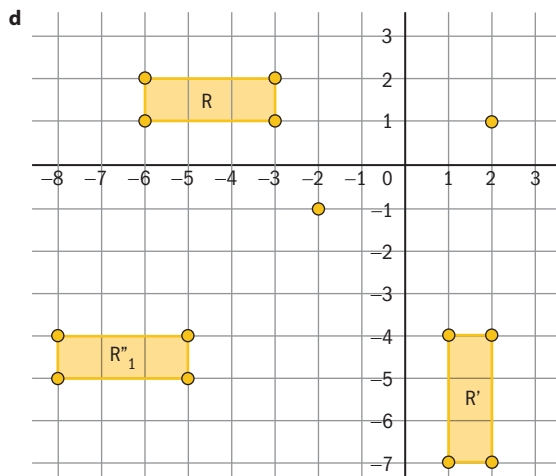
Translation by $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$

4 a Multiple answers possible:
Rotation of 180° around point (−3, 2).
Reflection in line $y = 2$ followed by a reflection in line $x = -3$.
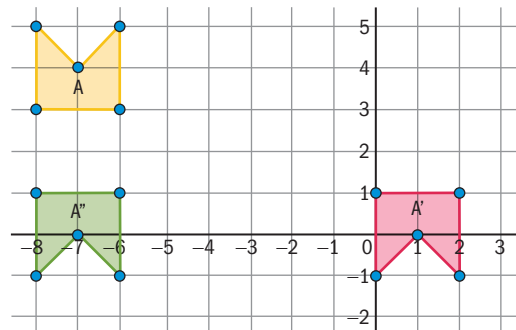
b Multiple answers possible:
Rotation 90° clockwise around point (2, 3).
Reflection in line $y = -x + 5$ followed by a reflection in line $x = 2$.
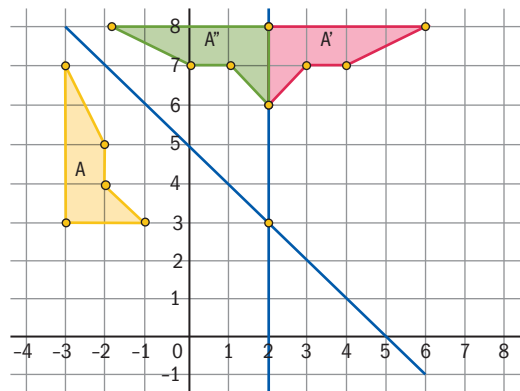
c Multiple answers possible:
Translation by $\begin{pmatrix} 8 \\ -8 \end{pmatrix}$.
Reflection in line $y = x + 3$ followed by a reflection in line $y = x - 5$.
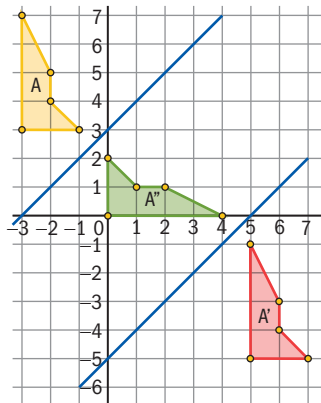
Mixed practice

1

Point	Translation by $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$	Rotation by 180º around the origin	Reflection in the x-axis	Reflection in the line $y = x$
(0, 0)	(–2, 5)	(0, 0)	(0, 0)	(0, 0)
(5, 3)	(3, 8)	(–5, –3)	(5, –3)	(3, 5)
(3, –2)	(1, 3)	(–3, 2)	(3, 2)	(–2, 3)
(–2, 5)	(–4, 10)	(2, –5)	(–2, –5)	(5, –2)
(–4, –3)	(–6, 2)	(4, 3)	(–4, 3)	(–3, –4)

2 a

b

c

d

e

f

g

h
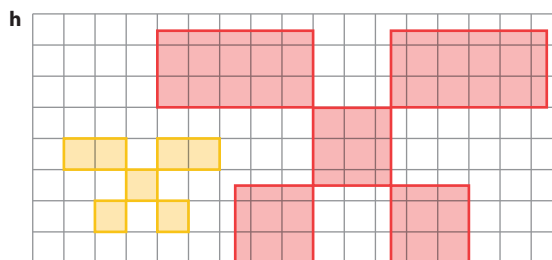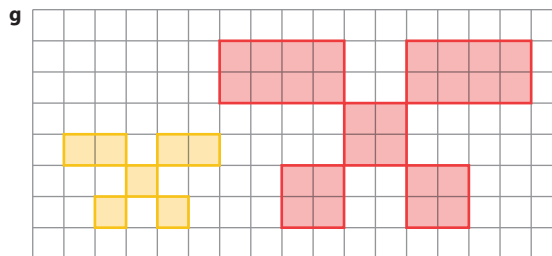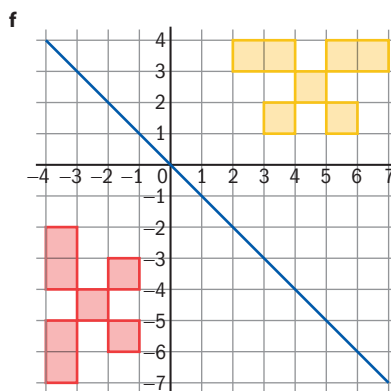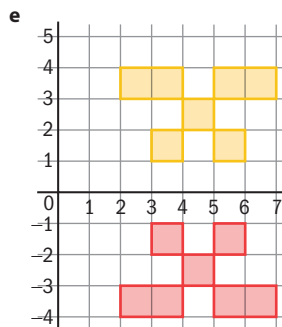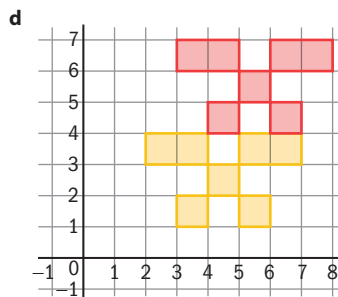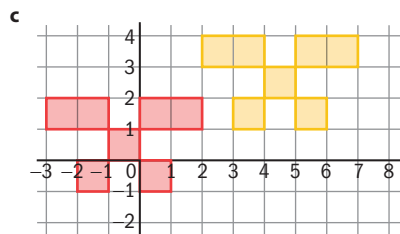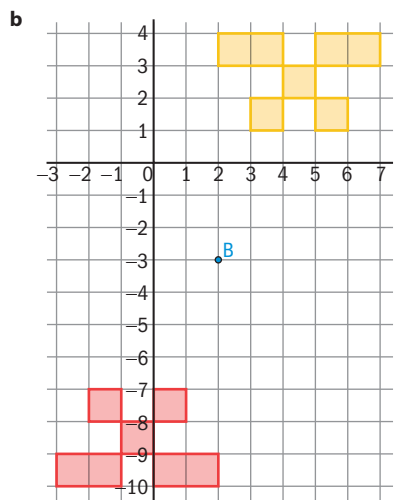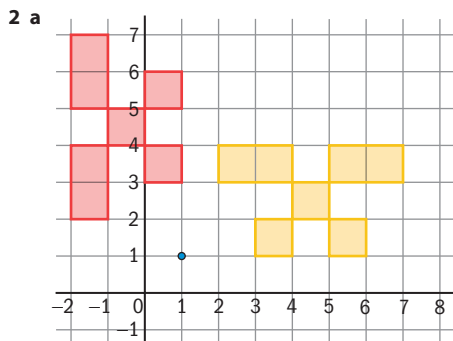
3 Different answers possible. One possible answer is this (going clockwise):

- Reflection in the line $y = 4 - x$.
- Rotation 90° clockwise around the point (4, 0).
- Reflection in the line $x = 4$.
- Rotation 180° around the point (4, 0).
- Reflection in the line $y = x - 4$.
- Rotation 90° counter-clockwise around the point (4, 0).
- Reflection in the line $y = 0$.

4 a 50 000
  b 0.5 km
  c 7 km
  d 7 cm
  e Because cities have an area, and the real-life distance is measured from the edge of one city to the closest edge of the next city. Whereas on a map, the cities may be represented by points at the city centers.

5 a Student's own drawing.
  b Since C is rotated about A to obtain D, CA and DA have the same lengths. The angle CAD being 60°, and the sum of all three angles in a triangle is 180°, the two other angles (ACD and ADC) must add up to 120°. These two angles are also equal, since CA and DA have the same lengths (property of an isosceles triangle). Since each of the three angles is 60°, the three sides are also of equal length, thus it is an equilateral triangle.

6 a Yes, it tessellates
  b A drawing similar to this one:
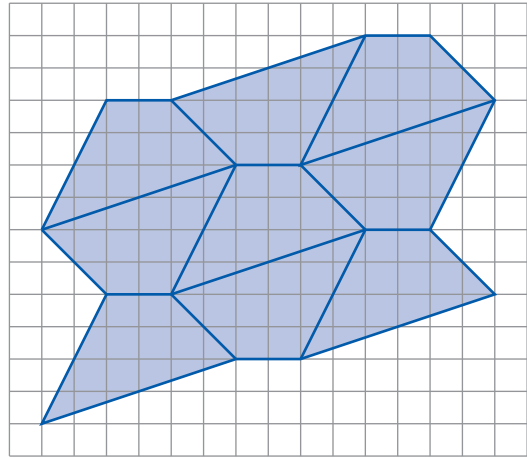
  c Each new tile can be obtained by rotating a tile 180° around the midpoint of one of its edges.

Unit 6 Answers

6.1

You should already know how to:

1 a 3, 5, 5, 10   **b** 19, 19, 19, 8

Practice 1

1 Quantitative, discrete   **6** Quantitative, continuous

2 Quantitative, continuous   **7** Quantitative, discrete

3 Qualitative   **8** Quantitative, continuous

4 Quantitative, discrete   **9** Quantitative, discrete

5 Quantitative, continuous   **10** Qualitative

Practice 2

1

stem	leaf
2	5
3	4 5 5 5 8
4	1 1 4 5 6 6 7 7
5	1 1 6 9
6	0 2
7	3

Key: 2 | 5 represents 25

2 a

stem	leaf
20	9
21	3 6
22	4
23	1
24	5
25	2 2 3
26	
27	2
28	
29	
30	5

Key: 21 | 6 represents 21.6

3 a i 4 km   **ii** 40 km   **b** 18   **c** 3   **d** 50%

4

stem	leaf
16	1 1 6 6 7 9
17	3 3 5 7 7
18	0 3 4
19	2 5 5 7

Key: 16 | 9 represents 169

Practice 3

1 49, 43, 28

2 30, 29, 30

3 a 11   **b** 63   **c** 42
d 70   **e** 46, 35
f Mathematics: since the range is smaller, data is more closely bunched.

4 a

stem	leaf
1	2 6 6 9
2	0 1 3 5 8
3	2 6 9
4	7

Key: 1 | 2 represents 12

b 4   **c** 16, 23, 35

Practice 4

1 a 730, 750, 670   **b** 360, 80

2 a 45.5, 51, 41   **b** 39, 10

3 37.2, 21.5, 12.9   **4** 12, 48, 13

5 a £70 000   **b** £48 000
c £32 000, £18 000   **d** £23 500, £11 500
e Senior employees earn more on average and the salaries are more spread out.

6 $x = 34$, $y = 46$, $z = 54$ **7** Student's own answers

Practice 5

1 a 10   **b** 20   **c** 9

2 a 47, 52, 61, 66, 75   **b** 28, 14   **c** 25

3 a 1, 2, 3, 4, 4   **b** 10, 13, 15, 18, 24
c 10 000, 10 300, 11 750, 12 250, 64 000

4

Number of lost property items

5 a 21   **b** 13, 27

c

Time (minutes)

6

Height, m

7 a 38   **b** 45.5   **c** 17

d

Number of diners

8

cm
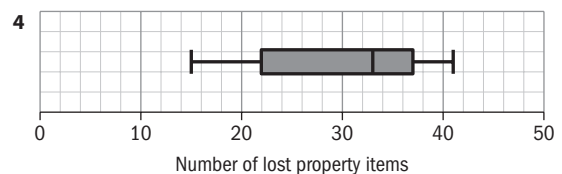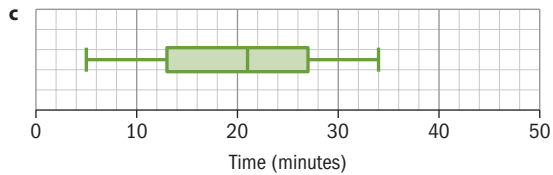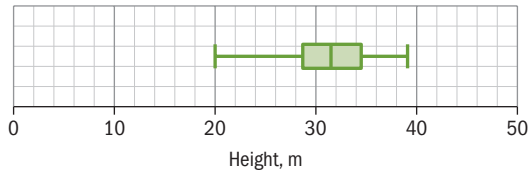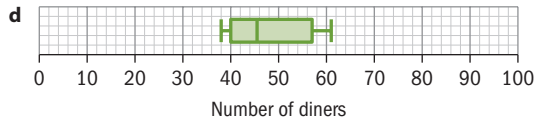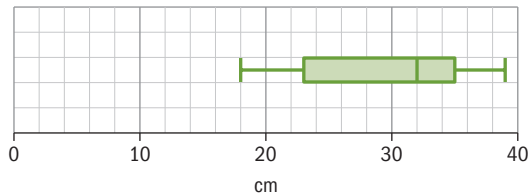
A = 8, B = 3, C = 2, D = 5, E = 9

Practice 6

1 a Med G: 38, Med B: 34; IQR G: 22.5, IQR B: 19.5

  b Both distributions are fairly symmetrical and unimodal. Boys a little more centered on the lower values.

  c On average, boys took less time and have less variation of times.

2 a

Santa Fe		Saint Paul
	2	6 9
	3	1
8 4 3	4	2 3
6 3	5	8 9
7 5	6	
8 4	7	1 3
6 3 3	8	0 2 5

Key: 3|4|2 represents 43 in Santa Fe and 42 in Saint Paul

  b There is no modal value for either city. In Sante Fe the temperatures are evenly distributed between 43 and 86 degrees.
In Saint Paul, the temperatures are are evenly distributed between 26 and 85 degrees.

c The temperature in Santa Fe is on average higher but less varied.

3 a

apple trees		pear trees
9 7 6 6 1 1	16	0 0 5 5 6 8
7 7 5 3 3	17	0 1 2 4 7 8
4 3 0	18	2 3
7 5 5 2	19	0 2 3 4

Key: 1 | 16 | 0 represents 161 for apple trees and 160 for pear trees

  b Both centered on lower values with a few taller trees. The apple trees are multimodal, the pear trees are bimodal.

  c Very similar. Slightly higher average for apple trees with a similar spread of heights.

4 Train B had a higher length if delay on average and a wider spread of delays.

5 a

Age
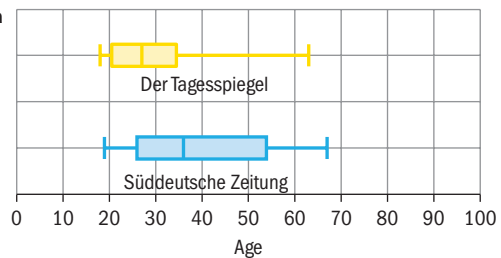
  b Süddeutsche Zeittung has on average older users but the ages are more varied.

6 Ray's sales are on average higher and they are also more varied.

Practice 7

1 a 28.4, 28, multimodal: 25, 31, 32 all have 2

  b Either mean or median – both fairly central. No clear mode.

2 a 28.9, 28   **b** 16.5   **c** 62

  d 54, outlier included since could be legitimate value.

  e Median – not affected by outliers.

3 a Boys: 60, 85

  b Girls: 65, Boys: 85; Outliers included since could be legitimate values.

4 a Male: 100 kg

  b Both sets symmetrical and unimodal

  c With the outlier excluded, both data sets are almost the same.

5 a

stem	leaf
1	2 5 6
2	1 4 4 5 5 5 5 6 6 8 9
3	0 1 3 5 6 8
4	2
5	
6	4
7	4
8	4 6
9	
10	7

Key: 1 | 2 represents 1.2%

Outliers at 6.4, 7.4, 8.4, 8.6, 10.7
b These may be the cantons that have large urban centers.
c Unimodal and centered on lower values.

Mixed practice

1 a Qualitative **b** Quantitative
c Quantitative **d** Quantitative

2 a Continuous **b** Discrete
c Continuous **d** Continuous

3 a

stem	leaf
4	6 8
5	2 9
6	0 1 4
7	0 2 5
8	2

Key: 4 | 6 represents 46

b 4 **c** 61 kg **d** 36 kg

4 a 39 min **b** 16 min **c** 32 min

5 a $r = 10$, $s = 13$ **b** 18

6 a 3, 17, 26, 32, 90
b

Mass (kg)
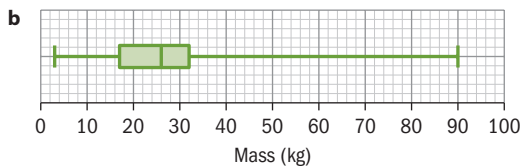
7 a 18 **b** $a = 10$, $b = 44$ **c** 40

8 a Film A had on average older people watching and a greater spread of ages.

b A: Young and old people attended.

9 a £132 000 **b** £560 000
c £559 000, included since a possible value
d Median: £85 000, not affected by outliers.

10 a

Farm A		Farm B
9	1	
	2	
	3	
9 9 8 8 5 3	4	9 9
7 6 5 5	5	0 4 4 6 7
5 4 4 3 0	6	2 4 6 8 8
	7	0 0 1 2

Key: 3 | 4 | 9 represents 43 for Farm A
and 49 for Farm B

b Farm B had a higher average and greater spread.
c Both look like Average/good hill because the range of values fits that profile.

Review in context

1 a

Percentage of land covered by forest
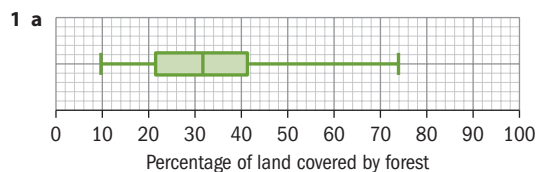
b Finland is a possible outlier.

2 2012 had a slightly lower average and values generally lower overall. Suggests forest areas reduced.

3 Student's own answers

6.2

You should already know how to:

1 4.2 and 6.0

2 a Discrete
b Continuous
c Discrete
d Continuous

3 Mean = 3.53 (3 s.f.)

4 a

b IQR = 5
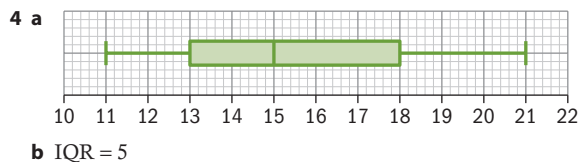
5 a $\frac{1}{4} = 25\%$ **b** $\frac{1}{2} = 50\%$ **c** $\frac{3}{4} = 75\%$

Practice 1

1 a $1.40 < x \le 1.50$ **b** $1.40 < x \le 1.50$

2 a

Time x (sec)	Frequency
$300 < x \le 350$	3
$350 < x \le 400$	4
$400 < x \le 450$	4
$450 < x \le 500$	2
$500 < x \le 550$	3
$550 < x \le 600$	2
$600 < x \le 650$	2

  b $350 < x \le 400$ and $400 < x \le 450$
  c $400 < x \le 450$

3 a Other ways of grouping the data are acceptable.

Height x (cm)	Frequency
$160 < x \le 170$	6
$170 < x \le 180$	6
$180 < x \le 190$	2
$190 < x \le 200$	4

  b $160 < x \le 170$ and $170 < x \le 180$
  c $170 < x \le 180$

4 a $a = 5$, $b = 17$, $c = 2$, $d = 27$
  b The 15th and 16th apples are both in this interval.

Practice 2

1 a

Time (T hours)	Frequency	Mid-interval value	Mid-interval value × frequency
$0 < T \le 10$	8	5	40
$10 < T \le 20$	12	15	180
$20 < T \le 30$	16	25	400
$30 < T \le 40$	11	35	385
$40 < T \le 50$	3	45	135
Total	50		1140

  b 22.8 hours

2 a 54   **b** $54 < w \le 57$ **c** 19   **d** $54 < w \le 57$

3 a 1.1 kg   **b** $1.0 < w \le 1.2$ **c** 51   **d** $1.0 < w \le 1.2$
  e 1 kg
  f The estimate of the mean would change slightly but the median would stay the same.

Practice 3

1 a 1.96
  b Upper quartile = 2.06, Lower quartile = 1.86
  c 0.2

2 a The three table entries are 24, 32, 36
  b
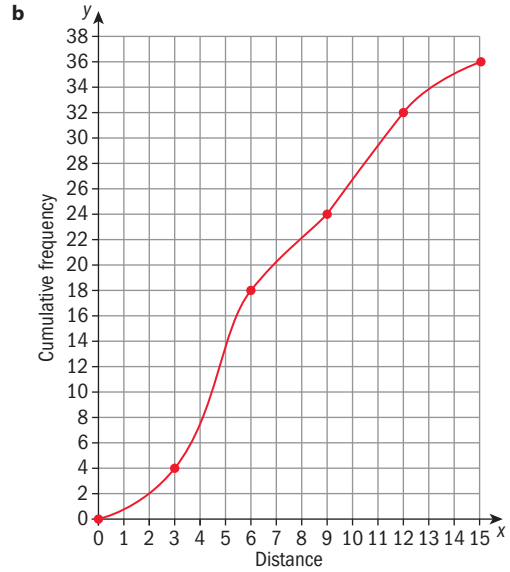
  c Lower quartile = 4.2, Median = 6, Upper quartile = 10.2
  d Range = 15, IQR = 6

3 a

  b 65, 72.25, 77.75, 83.5, 90
  c

4 a

b

0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7
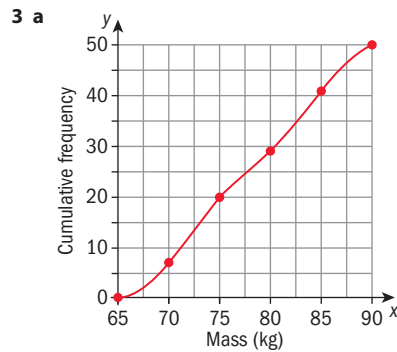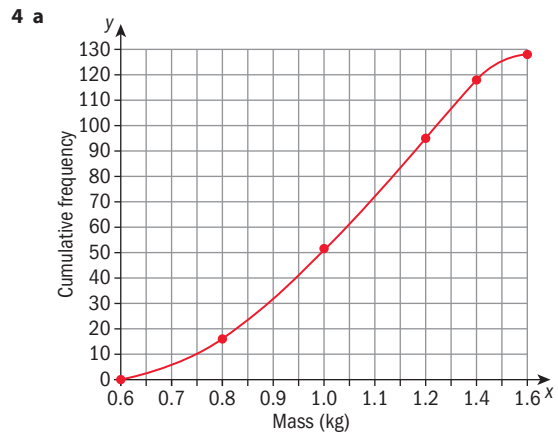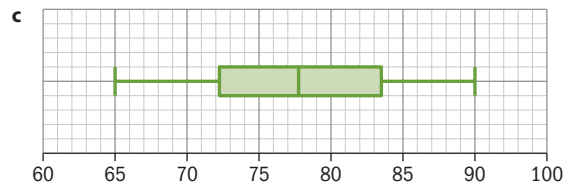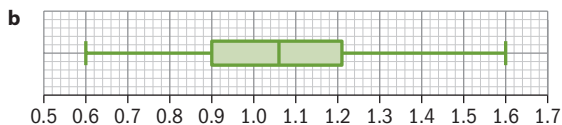
c i false **ii** false

d 1.2 kg (to the nearest 0.1 kg)

5 a
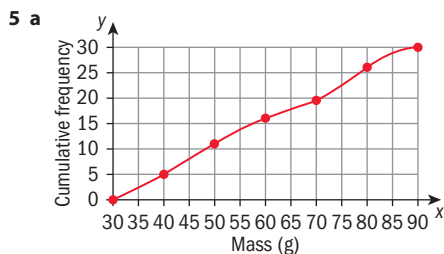
b 8 **c** 80; 90th percentile

6
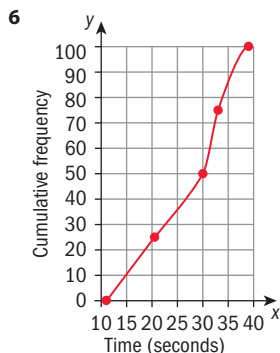
7 a 122 km/h

b 15th percentile is 104 km/h; 85th percentile is 130 km/h

c About 16%

d

85 90 95 100 105 110 115 120 125 130 135 140 145
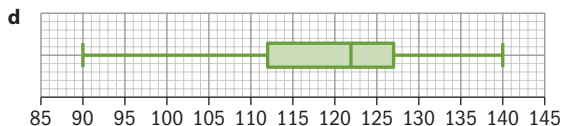
e On average, people drive faster on Motorway A.
The range of speeds is greater on Motorway B.

Practice 4

1 i a continuous, overlapping boundary **b** 10
values with continuous intervals

ii a discrete, no overlapping boundary **b** 5
values, gaps between intervals

iii a continuous, overlapping boundary **b** 3
values with continuous intervals

iv a discrete, no overlapping boundary **b** 20, 25
values, gaps between intervals

2 a 11.8
It makes sense, although the number of words in a
sentence must in reality be a whole number.

b $10 \le x \le 14$

c $10 \le x \le 14$

3 a 10.3

b There are two classes with a frequency of 6.

c $8 \le x \le 11$

d 19

4 a

Score	Frequency	Cumulative frequency
$0.5 \le x < 5.5$	1	1
$5.5 \le x < 10.5$	9	10
$10.5 \le x < 15.5$	15	25
$15.5 \le x < 20.5$	15	40
$20.5 \le x < 25.5$	5	45
$25.5 \le x < 30.5$	5	50
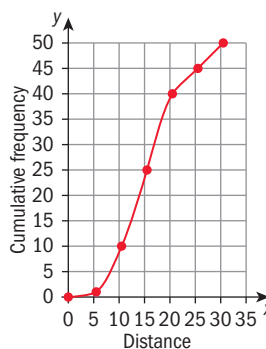
b 15.5

c Upper quartile = 19.5
Lower quartile = 11.5

d

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32

5 a $a = 20$, $b = 29$

b
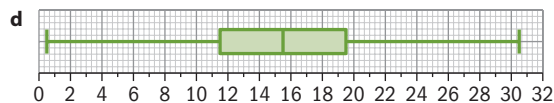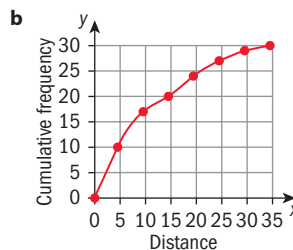
c Five point summary: 0, 3, 7.5, 17.5, 34

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36
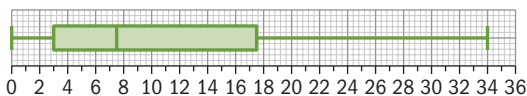
6 a

Age	Frequency	Cumulative frequency
$0 \le x < 10$	1	1
$10 \le x < 20$	3	4
$20 \le x < 30$	8	12
$30 \le x < 40$	7	19
$40 \le x < 50$	7	26
$50 \le x < 60$	1	27
$60 \le x < 70$	3	30

Other intervals are acceptable.

b

Cumulative frequency vs Age (years)
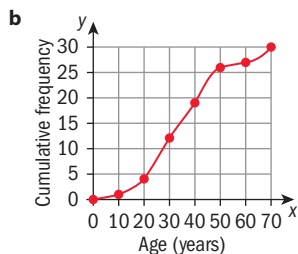
c 25–27% **d** 36 or 37%

7 a and **b** on same diagram

Box and whisker plots for Pedal cyclists and Pedestrians, scale 0 to 100
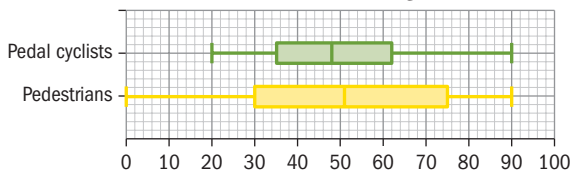
c On average, the pedestrians who were killed were older. There were also a wider range of ages of pedestrians who were killed.

Mixed practice

1 a

Mass (grams)	Frequency
$250 \le x < 300$	5
$300 \le x < 350$	7
$350 \le x < 400$	6
$400 \le x < 450$	4
$450 \le x < 500$	10
$500 \le x < 550$	3

Other intervals are acceptable.

b $450 \le x < 500$ **c** $350 \le x < 400$ **d** 398

2 a 60

b

Mark	Frequency	Cumulative frequency
$16 \le x \le 27$	9	9
$28 \le x \le 39$	21	30
$40 \le x \le 51$	18	48
$52 \le x \le 63$	23	71
$64 \le x \le 75$	19	90

c

Cumulative frequency vs Marks

3 a 26 cm **b** About 5.75 cm

c

Box and whisker plot, scale 10 to 40

4

Cumulative frequency vs Mass (grams)
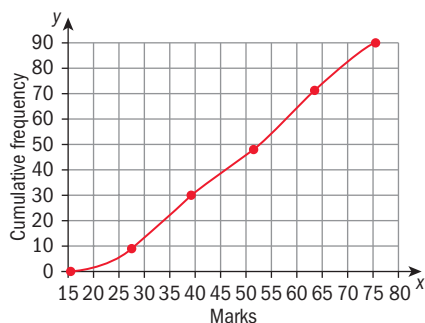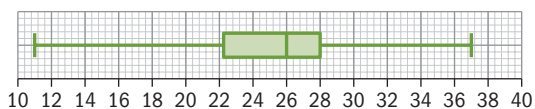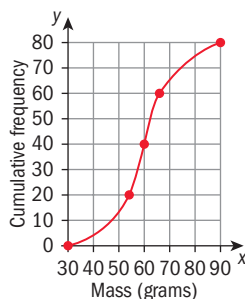
5 a 98.0 km/h to 1 d.p.

b $a = 167$ and $b = 277$

c

Cumulative frequency vs Speed (km/h)
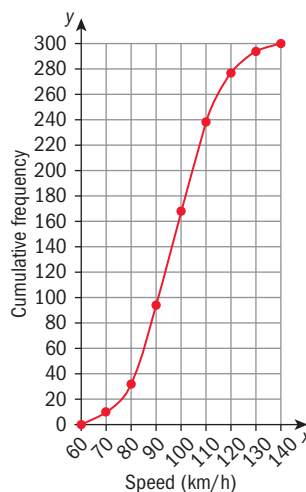
d 108 km/h

6 a i 72 grams **ii** 76 grams

b $x = 82$

7 a 24.2 cm

b About 11 cm

c 62 fish

d 76 small fish, 62 large fish

e About $1076

8 Any sensible comparisons.
e.g. the distributions are very similar.
The two box and whisker plots are the same.

Box and whisker plot, scale 0 to 100
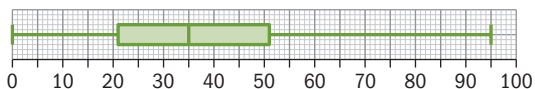
Review in context

1 a

Length L cm	Frequency	Cumulative frequency
$25 < L \le 27$	1	1
$27 < L \le 29$	5	6
$29 < L \le 31$	9	15
$31 < L \le 33$	21	36
$33 < L \le 35$	29	65
$35 < L \le 37$	19	84
$37 < L \le 39$	16	100

b

Length (cm)
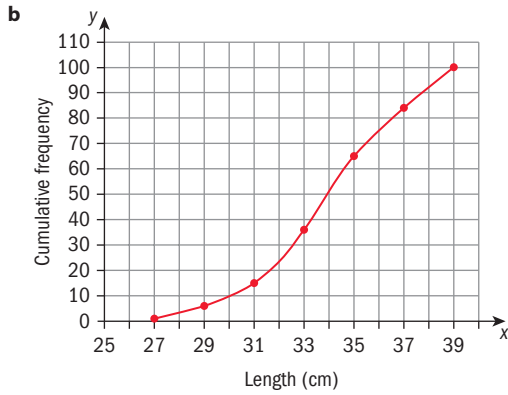
c 25, 32, 34, 36, 39   **d** 32
e **i** About 70
   ii About 56
   iii About 63%

6.3

You should already know how to:

1

Height	Frequency
$2.0 \le h < 3.0$	8
$3.0 \le h < 4.0$	5
$4.0 \le h < 5.0$	6
$5.0 \le h < 6.0$	5

2 Modal class is $2.0 \le h < 3.0$; Median class is $3.0 \le h < 4.0$;
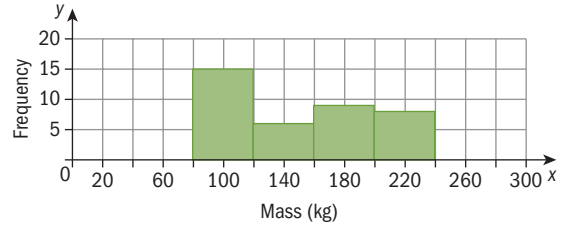Mean estimate = 3.83 cm (3 s.f.); Range estimate = 4.0 cm

3 a Quantitative   **b** Qualitative   **c** Quantitative

Practice 1

1

Height (cm)

2

Age (years)
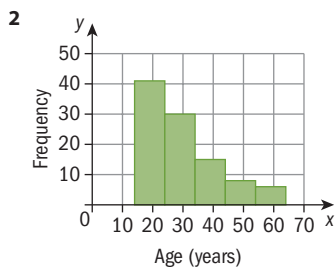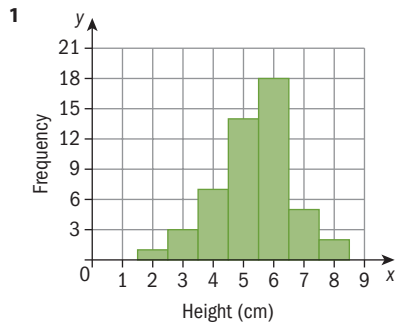
3

Class	Class boundaries	Frequency
80–119	79.5–119.5	15
120–159	119.5–159.5	6
160–199	159.5–199.5	9
200–239	199.5–239.5	8

Mass (kg)

Practice 2

1 a Discrete, ungrouped

b

Number of people
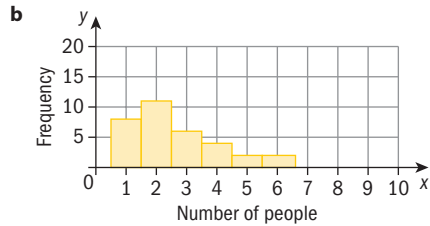
2 a

Number of eggs hatching	Frequency
1–4	2
5–8	2
9–12	6
13–16	9
17–20	3
21–24	3

b

Number of eggs

3 a Continuous

b

Length of leaf (cm)
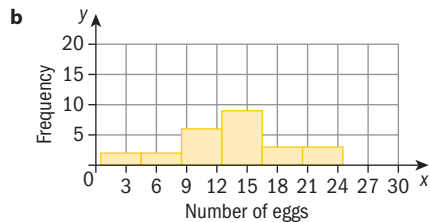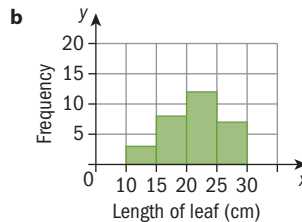
4 a 1–5
   b 0.5–5.5, 5.5–10.5, 10.5–15.5, 15.5–20.5, 20.5–25.5

c

y
20
15
Frequency
10
5
0   3 6 9 12 15 18 21 24 27 30 x
Distance (km)
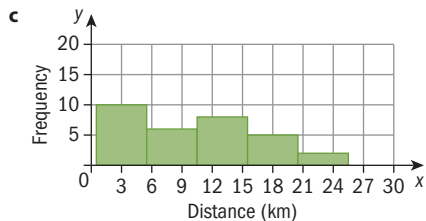
5 a Continuous
  b Student's own answer with continuous variable
  c No outliers, the data is evenly distributed within the range 1–5.

6 a

Time (h)	Frequency
$1.5 \le t < 2.5$	5
$2.5 \le t < 3.5$	6
$3.5 \le t < 4.5$	8
$4.5 \le t < 5.5$	7
$5.5 \le t < 6.5$	4

  b 3 hours 58 minutes

Practice 3

1 i Not symmetrical, multi-modal, contains possible outlier
  ii Not symmetrical, multi-modal, no outliers
  iii Symmetrical, unimodal, no outliers

2 a 36–40 cm   **b** 36–40 cm
  c 20 cm   **d** 38 cm
  e Relatively symmetrical, unimodal, no outliers

3 a 40
  b $158.25
  c Fairly symmetrical, unimodal, no outliers

4 a Discrete data
  b Student's own answers (bar chart with uniform distribution would be most likely – bars 2–3 units high.)

5 a

Mass (x)	Frequency
$10 \le x < 20$	3
$20 \le x < 30$	8
$30 \le x < 40$	23
$40 \le x < 50$	10
$50 \le x < 60$	4
$60 \le x < 70$	2

y
30
Frequency
20
10
0 10 20 30 40 50 60 70 80 90 x
Mass (kg)
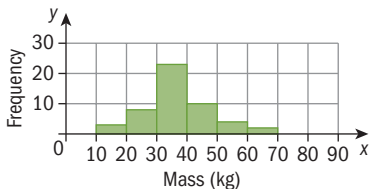
  b Symmetrical, unimodal, no outliers

Practice 4

1 a Different number of data points in each group
  b Men: $\frac{1}{6}, \frac{4}{15}, \frac{1}{3}, \frac{1}{6}, \frac{1}{15}$

  Women: $\frac{2}{25}, \frac{1}{10}, \frac{7}{50}, \frac{7}{25}, \frac{2}{5}$

  c

y   Men
0.35
0.30
0.25
Relative frequency
0.20
0.15
0.10
0.5
0   10 20 30 40 50 60 70 80 x
Time (min)

y   Women
0.45
0.40
0.35
Relative frequency
0.30
0.25
0.20
0.15
0.10
0.5
0   10 20 30 40 50 60 70 80 x
Time (min)
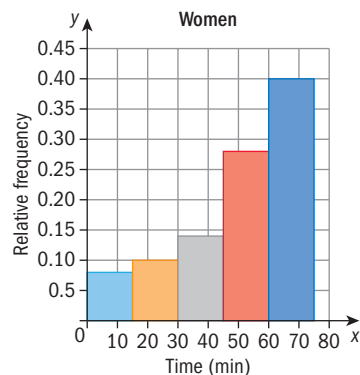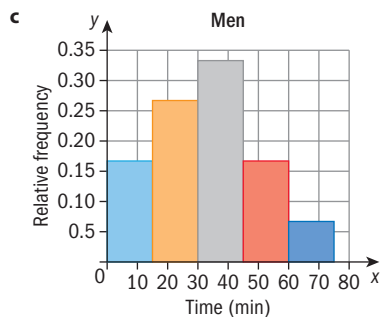
  d Men: symmetrical, unimodal; Women: unimodal but centered mainly on the longer times.
 **e** Women talked for longer on the phone than men. The spread of times is similar.

2 a Male: 9, 17, 17, 31, 26
  Female: 26, 42, 21, 11, 0 (nearest percent throughout)
  b

y   Male
50
40
Percentage
30
20
10
0   17 18 19 20 21 22 23 24 25 26 27 28 x
Mass (kg)

y   Female
50
40
Percentage
30
20
10
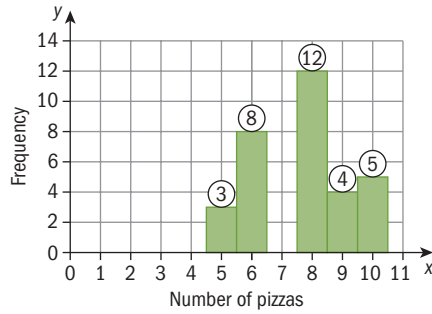0   17 18 19 20 21 22 23 24 25 26 27 28 x
Mass (kg)
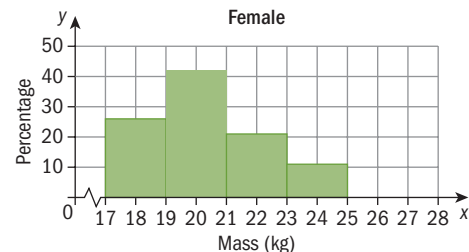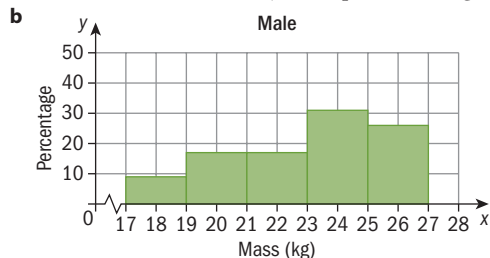
  c Male huskies tend to weigh more than female huskies. The spread is narrower for the female huskies as well. The distribution for the females is fairly symmetrical.

3 18

Mixed practice
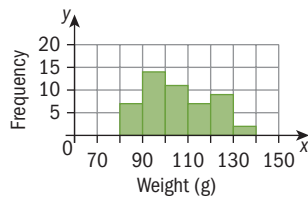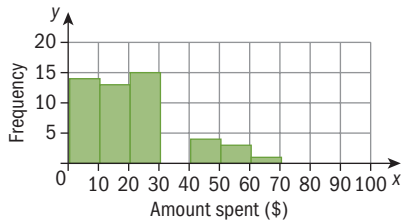
1 a The classes are of equal width

b
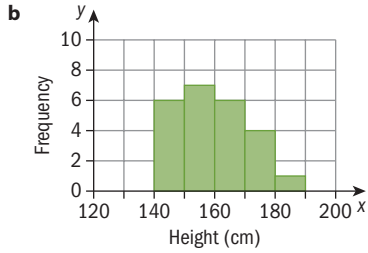
2

3 a

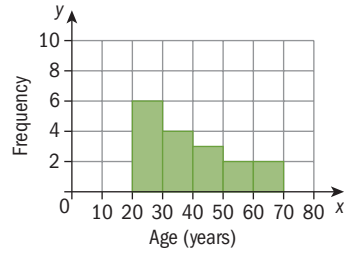| Weight (x)	Frequency
$80 \le x < 90$	7
$90 \le x < 100$	14
$100 \le x < 110$	11
$110 \le x < 120$	7
$120 \le x < 130$	9
$130 \le x < 140$	2

b

4 a

b 21–30

c 11–20

d Distribution is centered towards the lower amounts for eating out.

5 a 39.1 (3 s.f.)

b

c Skewed to the left, unimodal

6 a Different number of students in each group

b Both are unimodal, Sri Lanka is more symmetrical, Peru is skewed to the left.

7 a 72–80   **b** 77.6 cm

Review in context

1 a $(0 \times 4) + (1 \times 10) + (2 \times 7) + (3 \times 3) + (4 \times 1) = 37; \frac{37}{25} = 1.48$

b More children on average in Australia; Bigger spread of children in Australia. Both distributions unimodal and skewed slightly to the left.

2 a
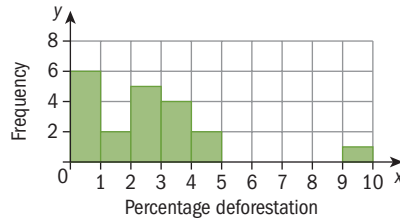
b Skewed to the left   **c** 9.2%

Unit 7 Answers

7.1

You should already know how to:

1 a 16 cm   **b** 16.2 cm   **c** 15.6 cm

2 $\sin A = \frac{a}{c}$; $\cos B = \frac{a}{c}$; $\tan A = \frac{a}{b}$; $\tan B = \frac{b}{a}$

3 a 5.77 m   **b** 42.0°

Practice 1

1 a 0.731   **b** 0.848   **c** 0.306
   d 0.0349   **e** 57.3   **f** 0.5
   g 1.03   **h** 0.0138
   i 1   **j** $\frac{\sqrt{3}}{3}$

2 a 27.8°   **b** 45°   **c** 45°
   d 26.1°   **e** 30°   **f** 60°
   g 82.9°   **h** 87.9°

Practice 2

1 a 13.77 cm   **b** 4.60 cm   **c** 5.5 cm
   d 11.92 cm   **e** 2.05 cm   **f** 7 cm

2 a 17.1°   **b** 45°
   c 50.0°   **d** 66.4°

3 a $AB = 25.6$ cm, $BC = 12.0$ cm, $\angle A = 28°$
   b $AC = 4.0$ cm, $BC = 3.0$ cm, $\angle A = 37°$
   c $AC = 2.3$ cm, $BC = 1.9$ cm, $\angle B = 50°$
   d $AC = 12.8$ cm, $BC = 5.7$ cm, $\angle B = 66°$
   e $AC = 3.2$ cm, $AB = 6.8$ cm, $\angle A = 62°$

Practice 3

1 $a = 5\sqrt{3}$ m, $b = 10$ m

2 $c = 6\sqrt{2}$ cm, $d = 6\sqrt{2}$ cm

3 $e = 2$ m, $f = 2\sqrt{3}$ m

4 $g = 6$ cm, $h = 6\sqrt{2}$ cm

5 $x = 60°$

Practice 4

1 56 m

2 24.0° (Depends on the accuracy of measurement and the height of the person)

3 115 m (3 s.f.)

4 37.6° (3 s.f.)

5 3 m, 5.20 m (3 s.f.)

6 93.0 m (3 s.f.)

Practice 5

1 a 35.5 cm (3 s.f.)   **b** 23.2 cm (3 s.f.)

2 a 15.7 mm (3 s.f.)   **b** 14.6 mm (3 s.f.)

3 44.0° (3 s.f.)

4 5.20 cm (3 s.f.)

---

5 A: 9.85 m B: 14.0 m C: 7.21 m (all the 3 s.f.)

6 367 m (3 s.f.)

7 a 308 km   **b** 073.7°

8 47.9 km (3 s.f.)   **9** 200 m

Mixed practice

1 South by 470 m

2 216°

3 116° 9.98 km east, 4.87 km south

4 17.0 m (3 s.f.)

5 a 145 m (3 s.f.)   **b** 12.7 cm (3 s.f)

6 a 24.6°   **b** $\sqrt{119}$ cm (10.9 cm to 3 s.f.)
   c 3.91 cm (3 s.f.)   **d** 5.91 cm (3 s.f.)
   e 122.7° (1 d.p.)

7 a 4.24 m (3 s.f.)   **b** 5.15 m (3 s.f.)

8 35.1°   **9** 73.9 m (3 s.f.)

Review in context

1 29 000 feet (3 s.f.)   **2** 12.6° (3 s.f.)

7.2

You should already know how to:

1 a 15.1 cm   **b** 18.1 cm

2 a 18.1 cm^2 **b** 26.0 cm^2

3 2.5 cm

Practice 1

1 18.5 cm (3 s.f.)   **2** 10.7 cm

3 12.1 cm   **4** 6.57 cm (3 s.f.)

5 106.3° (1 d.p.)

Practice 2

1 a Area $= 9\pi$ cm^2
     Circumference $= 6\pi$ cm
   b Area $= 400\pi$ mm^2
     Circumference $= 40\pi$ mm
   c Area $= 0.16\pi$ m^2
     Circumference $= 0.8\pi$ m

2 a Area $= 76.97$ mm^2
     Perimeter $= 35.99$ mm
   b Area $= 113.10$ cm^2
     Perimeter $= 42.85$ cm

3 a 7.40 cm   **b** 23.4 cm   **c** 34.8 cm   **d** 55.3 cm^2

4 a $\frac{14\pi}{3}$ cm   **b** $\frac{14\pi}{3} + 14$ cm   **c** $\frac{49\pi}{3}$ cm^2

5 a 86.64 m   **b** 410.50 m^2

6 a 197.92 cm   **b** 269.92 cm   **c** 3562.57 cm^2

7 17.55 cm   **8** 163.86 cm

9 62.83 cm^3 **10** 706.86 cm^3

Practice 3

1 a 110° **b** 80°

2 a 24.0° (3 s.f.) **b** 3.15 cm (3 s.f.)

3 a 200° (3 s.f.) **b** 14.0 cm (3 s.f.) **c** 22.0 cm (3 s.f.)

4 a 9.17 cm **b** 6.94 m

5 347.86 mm **6** 4.71 cm^2

7 radius = 7.9 cm **8** 3.00 cm (3 s.f.)

9 5.91 cm

10 a   **b** 323.78 m
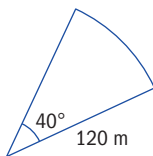
c 5026.55 m^2 **d** 82.08 m

Mixed practice

1 a 2.09 mm **b** 8.78 mm **c** 5.17 mm^2

2 a 62.3 km **b** 436 km^2

3 a Perimeter = 25.1 m, Area = 31.8 m^2.
 b Perimeter = 67.5 m, Area = 223 m^2.

4 a 3.93 m **b** 13.9 m **c** 68.7 m^2

5 a 120° **b** 16.8 cm
 c 32.8 cm **d** 13.9 cm

6 a 110° **b** 39.2 km **c** 16.4 km

7 a Student's own diagram:
   a correctly labelled rectangle.
 b Student's own diagram,
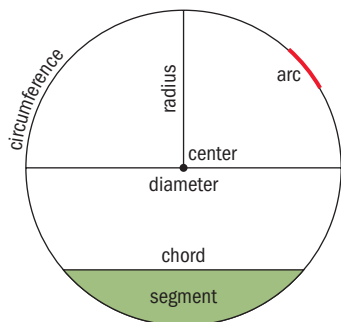   with center of the circle at the center of the rectangle.
 c 70.4 m^2
 d 12.0 %

8 30 cm

Review in context

1 a 23.6 cm **b** 19.1°

2 a 190.5 m^2 **b** 0.3473 m^2 **c** 190.1 m^2

3 a 12.86° **b** 16.16 m^2
 c 23.64 m **d** 15.43 m

7.3

You should already know how to:

1 a–g

Practice 1

1 a $a = 41°$, $b = 49°$
 b $c = 54°$, $d = 63°$
 c $e = 130°$, $f = 50°$
 d $g = 65°$, $h = 115°$
 e $j = 58°$, $k = 64°$
 f $m = 28°$, $n = 64°$

2 a $a = 50°$
   $\angle ACB = 90°$ (Thales' Theorem)
   $a = 180° − 90° − 40° = 50°$ (Angles in a triangle sum to 180°)
 b $b = 45°$
   $\angle DEF = 90°$ (Thales' Theorem)
   $\angle EFD = 45°$ (Base angles in an isosceles triangle)
   $b = 45°$ (Angles in the same segment)
 c $c = 48°$
   $\angle IKH =$ Angle $KHI = 76°$ (Base angles in an isosceles triangle)
   $\angle IJK = 104°$ (Opposite angles in a cyclic quadrilateral are supplementary)
   $\angle KIJ = 28°$ (Angles in the same segment subtended by equal chords are equal in size)
   $c = 180° − 28° − 104° = 48°$ (Angles in a triangle sum to 180°)
 d $d = 50°$
   $\angle SRP = 70°$ (Alternate segment theorem)
   $\angle PTR = 110°$ (Opposite angles in a cyclic quadrilateral are supplementary)
   $\angle STR = 180° − 110° = 70°$ (Angles on a straight line)
   $\angle SRT = 180° − 90° − 70° = 20°$ (Angles in a triangle sum to 180°)
   $d =$ Angle $SRP −$ Angle $SRT = 70° − 20° = 50°$
 e $e = 12°$
   $\angle UOV = 156°$ (The central angle is twice the inscribed angle)
   $e = \angle OUV = \angle OVU = \frac{180 − 156}{2} = 12°$ (Base angles in an isosceles triangle)
 f $f = 110°$
   $\angle BAE = 180° − 70° − 20° = 90°$ (Angles in a triangle sum to 180°)
   Therefore BE is a diameter of the circle and F is the center (Thales' Theorem)
   Hence $AF = FB$ and $\angle FAB = \angle FBA = 70°$ (Base angles in an isosceles triangle)
   $f = 180 − 70 = 110$ (Opposite angles in a cyclic quadrilateral are supplementary)

Practice 2

1 Draw a point D on the major arc between A and C.

$\angle ADC = \frac{1}{2} \times \angle AOC$ (The central angle is twice the inscribed angle)
$\angle ADC = 180° − \angle ABC$ (Opposite angles in a cyclic quadrilateral are supplementary)
Now equate the two expressions for Angle ADC.

$\frac{1}{2} \times \angle AOC = 180° − \angle ABC$

Therefore $\angle ABC = 180° − \frac{1}{2} \times \angle AOC$

2 Draw in the line OQ.
 $\angle OYQ = \angle OXQ = 90°$
 Consider the quadrilateral $OXQY$.
 $\angle XOY = 2 \times \angle XPY$ (The central angle is twice the inscribed angle)

$\angle XOY = 360° - 90° - 90° - \angle XQY$ (Angles in a quadrilateral sum to 360°)

Equating the two expressions for $\angle XOY$ gives:

$2 \times \angle XPY = 180° - \angle XQY$

$\angle XPY = \frac{1}{2}(180° - \angle XQY)$

3 $\angle BDE = \angle CBE$ (Alternate Segment Theorem)
$\angle BED = \angle CBE$ (Alternate angles)
Therefore $\angle BDE = \angle BED$ and triangle BDE is isosceles.

4 $\angle BAC = \angle BDC$ (Angles in the same segment)
Both triangles share the angle at X.
Therefore $\angle DBX = \angle ACX$
(If the two triangles share two common angles, their other angles will be equal)
Triangles ACX and DBX are therefore similar.

5 $\angle XEY = \angle YEZ$ (Angles in the same segment subtended by equal chords are equal in size)
$\angle EYZ = \angle EYX = 90°$ (lines BE and DF are perpendicular)
Therefore triangles EXY and EZY are both right-angled triangles with $\angle XEY = \angle ZEY$.
Since $\angle XEY$ is complementary to $\angle EXY$ and $\angle ZEY$ is complementary to $\angle EZY$
$\angle EXY = \angle EZY$.

Mixed practice

1 a $a = 25°$, $b = 40°$ **b** $c = 84°$

2 a $a = 23°$ **b** $b = 132°$

3 a Converse: If $3a - 2 = 19$, then $a = 7$. Both statement and converse are true.
  b Converse: If $a - b = 0$, then $a = b$. Both statement and converse are true.
  c Statement is true (as wingless Moa bird is now extinct.) Converse: If it has wings, then it is a bird is false. (e.g. it could be an insect.)
  d Converse: If a polygon is a square, then it has four sides is true. The statement is false (e.g. it could be a rectangle) but the converse is true.
  e Converse: If $c^2 = 81$, then $c = 9$. The statement is true but the converse is false (c could be –9).
  f Converse: If a right-angled triangle's hypotenuse is the diameter of a circle, then its vertices will be on the circumference of the circle. Both statement and converse are true.

4 $\angle ADC = 180° - \angle ABC$ (Opposite angles in a cyclic quadrilateral are supplementary)
Since $\angle ABC = \angle DAB$, $\angle DAB + \angle ADC = 180°$
Therefore AB and DC are parallel.
$\angle BAC = \angle ACD$ (alternate angles)
Therefore $AD = BC$ (Equal angles are subtended by equal chords)

5 Draw in lines OA and OC.
$\angle OCB = \angle OBC$ (Base angles in an isosceles triangle)
$\angle OAB = \angle OBA$ (Base angles in an isosceles triangle)
Since $\angle OBA = \angle OBC$, all four of these angles are equal.
$\angle OAC = \angle OCA$ (Base angles in an isosceles triangle)
So therefore the two base angles in triangle ABC are equal, since $\angle OAC + \angle OAB = \angle OCA + \angle OCB$.
Therefore triangle ABC is isosceles.

Review in context

1 44°

2 4.94 m

7.4

You should already know how to:

1 $(2x - 3)(x + 1) \Rightarrow x = \frac{3}{2}$ or $x = -1$

2 12 cm

Practice 1

1 a $x = 8$ **b** $x = 6$ (6, 12)
  c $x = 7$ (6, 8)   **d** $x = 8$ (5, 8)
  e $x = 9$ (6, 8, 9, 12)   **f** $x = 3$ (1, 2, 3) or $x = 4$ (2, 3, 4)

Practice 2

1 a $x = 3$
  b $x = 5$ (2, 3, 4)
  c $x = 2$ (2, 3, 4, 12)

Mixed practice

1 a Student's own sketch
  b AX and BX take values 1 and 10 or vice versa.
    CX and DX take values 2 and 5 or vice versa.

2 a $x = 5$ **b** $x = 9$
  c $x = 2$ **d** $x = 1$, $y = 4$
  e $x = 18$ **f** $x = 5$

3 $x = 8$, $y = 4.5$, $z = 7.5$

4 $x = 3.6$, $y = 6.4$

5 $x = 9$, $y = 12$

6 a Student's own demonstration.   **b** $x = 4$, $y = 3$

Review in context

1 a Student's own sketch.
  b Student's own demonstration.
  c 25.25 cm

2 a 370 m
  b 381 m
    They are very similar.

Unit 8 Answers

8.1

You should already know how to:

1 a 10, 12   **b** 24, 29

2 a 27   **b** −13

3 a $n = 4$ **b** $n = 3.2$

4 a $u_n = 4n$ **b** $u_n = 12n$ **c** $u_n = -3n$

Practice 1

1 a 3, 7, 11, 15, 19   **b** 8, 26, 56, 98, 152
  c 9.75, 9, 7.75, 6, 3.75   **d** 0, 2, 6, 14, 30

2 47

3 It is not; all terms of the sequence are odd.

4 The 22nd term

5 $9\frac{1}{3}$

6 18 (Note: the first term of the sequence is u_0.)

7 a, iv   **b**, v   **c**, vi   **d**, i   **e**, iii   **f**, ii

Practice 2

1 a 1, 3, 11, 43, 171
  b 2, 5, 11, 23, 47
  c −4, 3, 10, 17, 24
  d 3, 2.5, 2.25, 2.125, 2.0625

2 −3, 6, −3, 6

3 a 2, 0.5, −1, 2, 0.5, −1
  b The numbers 2, 0.5, −1 keep repeating.
  c 2, 0.5, −1, 2, 0.5, −1

4 The 9th term   **5** 6562

Practice 3

1 a $a_n = 8n$ **b** $b_n = 3n + 11$ **c** $c_n = -14n$

  d $d_n = 60 - 11n$ **e** $e_n = 18n - 7$ **f** $f_n = \frac{1}{2}n + 16\frac{1}{2}$

  g $g_n = 9 - \frac{2}{3}n$ **h** $h_n = 4.6n - 6$ **i** $i_n = 33 - 18n$

2 a $a_n = 2n - 1$ **b** $b_n = 50 - 3n$

  c $c_n = \frac{1}{2}n + 9$ **d** $d_n = 4n - 13$

3 a Because the distance there and back from the plug to the first bulb is 2×250 cm = 500 cm.
    However, between the bulbs is only 10 cm, adding 2×10 cm = 20 cm to each new bulb and back.
  b $d_n = 500 + 20n$

4 a 6 + 14 + 14 = 34
  b 6 + 28n
  c 14

5 a 690
  b $7n - 10 = 102$
        $7n = 112$
           $n = 16$
    The 16th term

6 $u_n = 5n - 6$

7 a 9   **b i** 16   **ii** −56

8 47   **9** −2

Practice 4

1 a $165
  b $103.75
  c 3 hours 45 minutes.
  d $35 per hour

2 a $y = 1.2x + 10$
  b €12.88
  c 6.7 Gb

3 a EZ-framers $y = 0.14x + 16$
    Perfect pictures $y = 0.18x + 10$
  b EZ-framers €24.40, Perfect pictures E20.80; EZ-framers charge more
  c Edge length of 150 cm

4 a $h = 3 + 0.5t$
  b 6 m
  c 6.75 m
  d The tree is unlikely to keep growing indefinitely. The rate is not likely to be consistent throughout the year.

Mixed practice

1 a $u_n = 4n + 2$
  b $u_n = 49 - 7n$
  c $u_n = n^2 + 1$
  d $u_n = n^2 + 2n$
  e $u_n = 4.5n - 2$
  f $u_n = 2.5n^2 - 3n$
  g $u_n = -2n^2 + 10n + 1$
  h $u_n = -0.5n^2 + 2.5n + 4$

2 311

3 a 12, 7, 2, −3, −8
  b $u_n = 17 - 5n$

4 a 15, 22, 29, 36, 43
  b $u_n = 7n + 15$
    $u_{10} = 7 \times 10 + 15 = 85$

5 1008

6 a 7   **b** 34

7 a constant difference of 6, therefore a linear sequence
  b $T_n = 6n + 2$
  c 9

8 a $y_r = 10 + 1.2x$; $y_g = 16 + 0.8x$; $y_b = 6 + 2x$
  b When $x = 5$, $y_r = y_b = 16$
  c When $x > 15$, i.e. if a customer uses more than 15 gigabytes of data in a month.

Review in context

1 a $u_n = 6n + 6$ **b** 186

2 a constant difference of 70, therefore a linear sequence
  b $P_n = 70n - 30$
  c 530
  d e.g. Because a greater proportion of a one carriage train is taken up by the place where the driver sits, engine, or service areas.

8.2

You should already know how to:

1 a $\frac{1}{4}$ **b** $\frac{2}{5}$

  c $\frac{1}{x}$ **d** $\frac{1}{20}$

2 a $x = 3$ **b** $x = 3$ **c** $x = 4$

3 a $\frac{5}{3}$ **b** $\frac{1}{2}$ **c** $\frac{45}{133}$ **d** $\frac{1}{6}$

4 a $f(h) = 10h$ **b** $c(m) = 4.5m + 2$

5 a
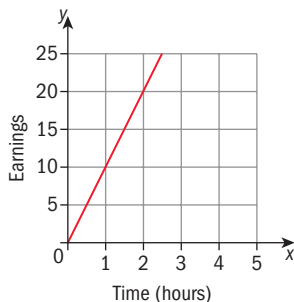
b
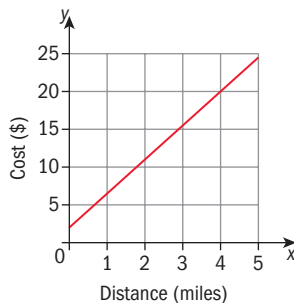
6
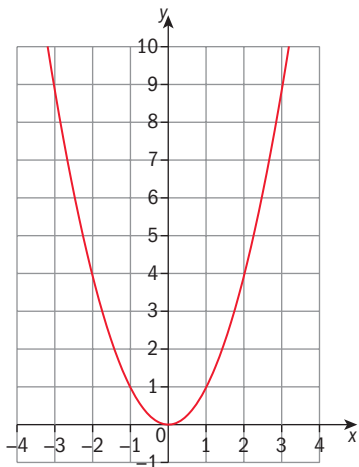
Practice 1

1 a $x = \frac{y+4}{5}$ **b** $x = \pm\sqrt{3(y-3)}$

  c $m = p^2 - 2$ **d** $w = \frac{5 \pm \sqrt{V}}{2}$

  e $V_i = \pm\sqrt{V_f^2 - 2ad}$ **f** $a = \frac{V_f - V_i}{t}$

  g $R = \frac{V}{I} - r$ **h** $r = \frac{mV^2}{F}$

  i $V = \pm\sqrt{\frac{Fr}{m}}$ **j** $m = \frac{F}{\frac{1}{2}V^2 + gh}$

  k $M = \pm\frac{3RT}{\mu^2}$ **l** $a = \frac{1}{3}\left[\pm\sqrt{c+1} - 2b\right]$

2 $A = \pi r^2; r = \sqrt{\frac{A}{\pi}}$ **3** $r = \sqrt{\frac{3V}{\pi h}}$

4 $t = \sqrt{\frac{2d}{a}}$ **5** $L = g\left(\frac{T}{2\pi}\right)^2$

6 $r = \sqrt{\frac{kq_1 q_2}{F}}$

Practice 2

1 $c(b) = kb$

  a c is doubled   **b** b is tripled

  c c is multiplied by $\frac{2}{3}$ **d** $4k$ is added to c

2 a $v(t) = kt$ **b** $k = 8$ **c** $v = 200$

3 $V = 2$

4 0, 8.54, 9.8

5 a $k = 1.6 \div 2 = 2.4 \div 3 = 5.6 \div 7 = 7.2 \div 9$

  b $k = 0.8$

  c 4 kg

6 $a : k = 3, c : k = \frac{1}{2}, d : k = \frac{4}{5}, f : k = -4$

Practice 3

1 a i Direct proportion: the perimeter varies directly as the length of the sides of the triangle.

    ii $k = 3$

  b i Direct proportion: the area that can be painted varies directly as the number of liters of paint.

    ii $k = 0.2$

  c i Direct proportion: y varies directly as x.

    ii $k = 4$

  d i Inverse proportion: y increases proportionally as x decreases, and vice versa; y varies inversely as x.

    ii $k = 3$

  e i Direct proportion: y varies directly as x.

    ii $k = \frac{1}{5}$

2 a $f(v) = \frac{k}{v}$ **b** $k = 24$ **c** $f = 12$

3 $y = 6.5$

4 $T = 2.32$ (3 s.f.)

5 1.6, 0.667 (3 s.f.)

6 a $d(w) = \frac{k}{w}$ **b** $k = 30$ **c** 24 kg

7 a $k = 1 \times 300 = 4 \times 75 = 8 \times 37.5 = 10 \times 30 = 20 \times 15$

  The cost per person halves when the number of people doubles, so this is an inverse relationship.

  b $k = 300$

  c \$12

8 a Yes. Direct proportion.

  $k = 390 \div 1.5 = 624 \div 2.4 = 1274 \div 4.9 = 2158 \div 8.3 = 260$.

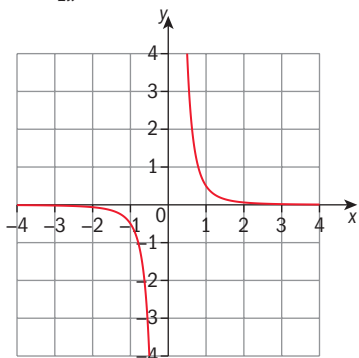 The increase in tiger population varies directly as the amount spent on conservation.

  b Increase in population = $260 \times$ Amount spent

  c \$12.3 million (3 s.f.)

9 a i Yes, time (in seconds) to see the lightning varies directly as the distance (in m) $\left(k = \frac{1}{300\,000\,000} \right)$.

  ii Yes, time (in seconds) to hear the thunder varies directly as the distance (in m) $\left(k = \frac{1}{340} \right)$

  iii Yes, the time to hear the thunder varies directly as the time to see the lightning $\left(k = \frac{300\,000\,000}{340} = \frac{15\,000\,000}{17} \right)$

b i 0.000 016 7 seconds (3 s.f.)

  ii 14.7 seconds (3 s.f.)

Practice 4

1 a i direct variation, $y = kx^3$ and k is constant ($k = 1$)

  ii $y = x^3$

b i inverse variation, $y = \frac{k}{x}$, k is constant ($k = 24$)

  ii $y = \frac{24}{x}$

c i direct variation $y = kx^2$, k is constant ($k = 5$)

  ii $y = 5x^2$

d i inverse variation, $y = \frac{k}{x}$, k is constant ($k = 60$)

  ii $y = \frac{60}{x}$

e neither, the first three table entries follow the rule $y = \frac{1}{x}$ but the last one doesn't.

f neither, this sequence follows no recognisable pattern

2 a $y = 48$　　　　**b** $y = 96$

  c $y = 6$　　　　**d** $y = 3$

3 a $x = \frac{5}{12}$

  b $x = 2.18$ (3 s.f.)

  c $x = 17.3$ (3 s.f.)

  d $x = 11.4$ (3 s.f.)

4 a 16π cm^2　　　　**b** $A = 4\pi r^2$

  c The surface area increases by a factor of 25

  d The radius is multiplied by a factor of $\frac{1}{\sqrt{2}}$

5 691 N (3 s.f.)

6 3.0 % to the nearest 0.1 %

7 5.93 cm (3 s.f.)

8 2.58 cm (3 s.f.)

Practice 5

1 a $y = 6x$

b $y = \frac{12}{x}$

c $y = 7.5x^2$

d $y = \frac{80}{x^2}$

e $y = 7.8125x^3$

f $y = \dfrac{1}{2x^3}$

2 a Neither, because the line does not pass through the origin.

b Direct, as x increases y increases. y is directly proportional to a positive power of x.

c Neither, the graph crosses the x-axis but no power of x with a non-zero coefficient can give a y-value equal to 0.

d Direct, y varies directly as x^2, with a negative proportionality constant k.

e Direct, y varies directly as x, with a negative proportionality constant k.

f Direct, y varies directly as a higher power of x, with a negative proportionality constant k.

3
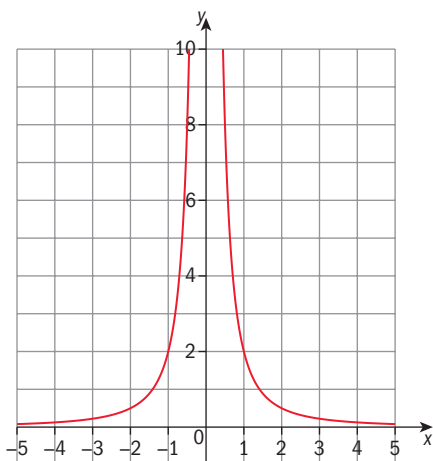
a y can't have a negative value because $2x^2$ can take only positive values.

b y approaches zero

c y becomes extremely large

d Because y becomes smaller as x becomes larger, and vice versa. x varies inversely as y.

Mixed practice

1 $r = \sqrt[3]{\dfrac{3V}{4\pi}}$

2 $T = \dfrac{Pv}{500}$

3 $r = \sqrt{\dfrac{Gm_1 m_2}{F}}$

4 $C = \dfrac{2W}{U^2}$

5 $c = \sqrt{\dfrac{E}{m}}$

6 $c = \dfrac{v\left(1 + \left(\dfrac{v}{v_0}\right)^2\right)}{\left(\dfrac{v}{v_0}\right)^2 - 1} = \dfrac{v\left(v_0^2 + v^2\right)}{v_0^2 - v^2}$

7 a direct relationship
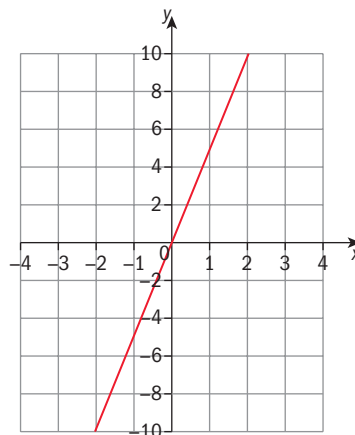
b direct relationship
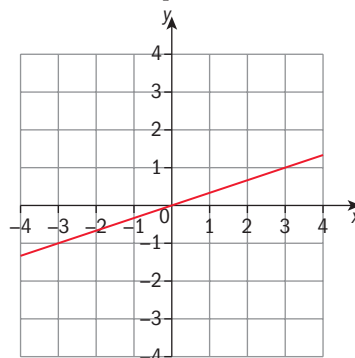
c direct relationship
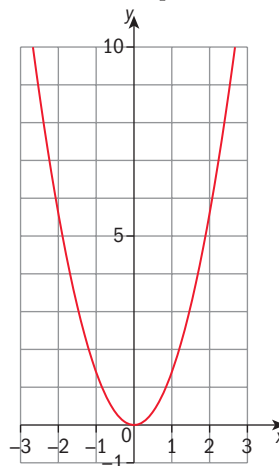
d inverse relationship
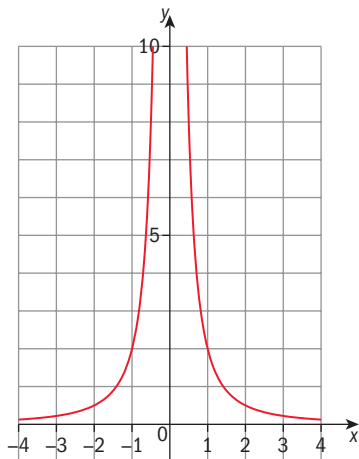
e inverse relationship
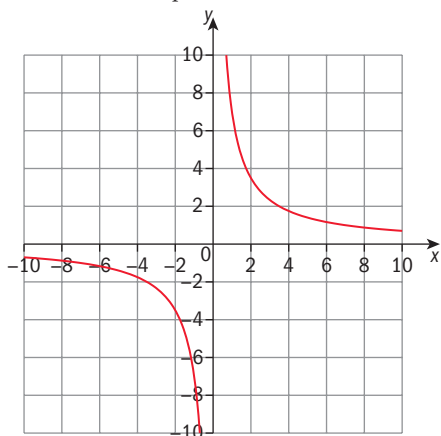
f inverse relationship
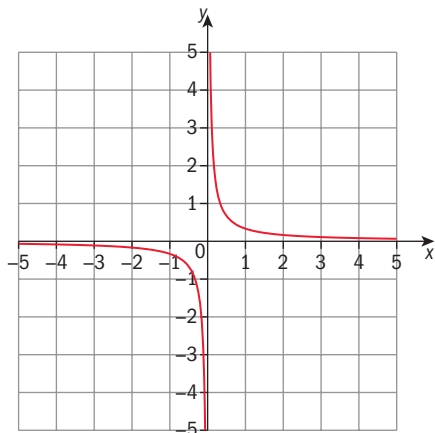
g direct relationship
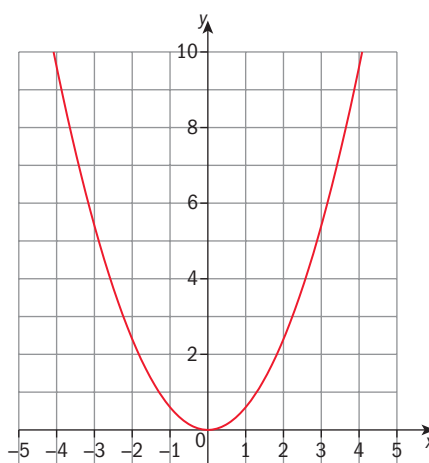
h inverse relationship
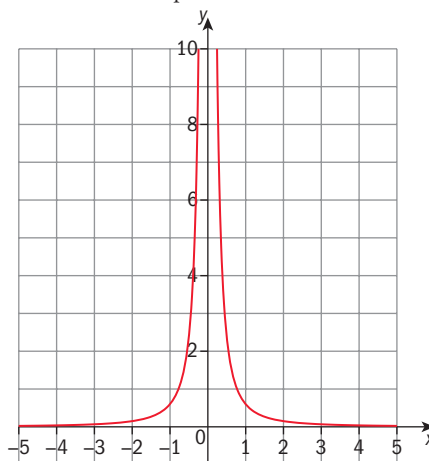
8 a $k = 0.75$ **b** $x = 0.75t$ **c** $x = 6.15$

9 $P = 2$ **10** $c = \pm 3.5$ **11** $p = 25$, $q = 0.2$

12 a $y = \dfrac{x^2}{4}$ **b** $y = -\dfrac{5}{x}$

13 a The more bottles are bought, the higher the cost.
  Cost $= 1.3 \times$ number of bottles, direct proportion
  b No, because the cost for 2 bottles or 3 bottles is the
    same, £2.60. The cost does not increase proportionally to
    the number of bottles bought.

14 360 N

15 $N =$ the whole number part of $\dfrac{3k}{4\pi r^3}$

16 70.4 kg **17** 367.2 m **18** 500

19 a $V = kr^2$, $k = \pi h$ **b** $V = kh$, $k = \pi r^2$

Review in context

1 a 830 km/h **b** 900 km/h **c** $(870 + x)$ km/h

  d $(870 - y)$ km/h **e** $t = \dfrac{d}{V}$

  f 6.39 hours $=$ 6 hours 23 minutes 27 seconds
  g 6.70 hours $=$ 6 hours 42 minutes (6 hours 41 minutes
    56 seconds)
  h 6.18 hours $=$ 6 hours 10 minutes 40 seconds

i 70 km/h (headwind)

j 50.3 km/h (tailwind – 3 s.f.)

2 Alphatown $62 500
Boomcity $125 000

3 a

Pipe diameter (cm)	10	20	30	40	50
Number of houses	50	200	450	800	1250

b Between 300 and 350 houses

c
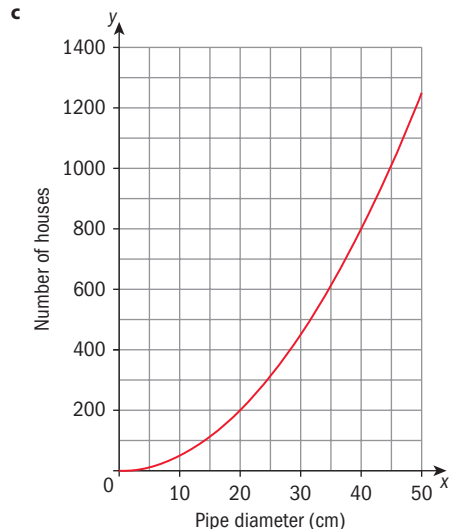
d Between 310 and 340 houses

e 312 houses

f Student's own answer

g Student's own answer. A possible argument could be to find ways to reduce household consumption so that a pipe can serve more houses without needing to be enlarged.

4 Moving the heater to B reduces the heat received by 36%.

5 a Temperature change –0.71°C

b Diameter of trees 2.24 m

6 a 1220 pairs of Classic jeans

b They should sell Classic jeans at $113 per pair to double their sales.

7 a $2.5 \div 3^3 \neq 5 \div 5^3$, there is no constant proportion between the cube of the radius and the price.

b One large scoop would cost $11.57

c Student's own suggestions. Possible arguments could include that the price of an ice-cream should cover other costs, such as the salary of the waiter, the cost of the equipment to make ice-cream, etc. These costs can be covered in the first scoop or two, which will therefore be more expensive than each additional scoop.

Unit 9 Answers

9.1

You should already know how to:

1
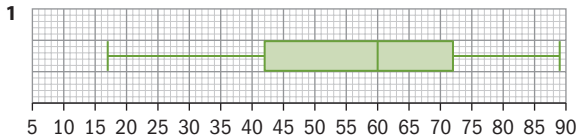
stem	leaf
1	7 9
2	9
3	3 4
4	
5	0 3 4 5 5
6	5 7
7	0 1 1 3 5 7
8	8 9

2
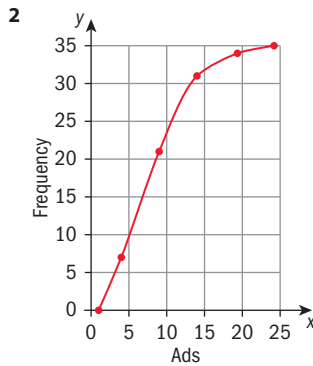
3 Mean = 1.667 to 3 d.p.
Median = 1.69
Range = 0.32
Lower quartile = 1.58
Upper quartile = 1.73
Interquartile range = 0.15

Practice 1

1 a You don't want to break all the eggs because then there would be none left to eat.
   b Either number the eggs and use a random number generator to choose which eggs to break open, or place the eggs in a large bag and take them without looking.

2 a Use a random number generator to generate 20 random numbers between 1 and 80.
   Choose the corresponding apartments.
   b Because the apartments with numbers that are 4 apart might all share some characteristic that would influence the number of people who live in them. For example, if there are four units per floor, the first apartment on each floor might happen to be the largest.

c The different types of apartment within the building.

3 a A systematic sample.
   b It may not be suitable, as he may not only have his friends on his contacts list.
   c 83.3%

4 a Number the employees from 1 to 900.
   Use a random number generator to generate 40 random numbers between 1 and 900.
   Choose the corresponding employees.
   b Number the employees from 1 to 900.
   Use a random number generator to pick a random starting point. Choose every 20th employee from that point.
   c Take a random sample of 1 man and 2 women from pay grades A and B combined; 4 men and 3 women from pay grade C; 8 men and 32 women from pay grade D; 24 men and 16 women from pay grade E.
   There are too few people on pay grade A for it to be considered separately.

5 a Yes. The school will have a register of all students.
   b No. There is no complete list of the supporters of a team.
   c Yes. The council keeps an electoral roll, which is a sampling frame for all voters.
   d Yes, provided it was possible to access the government or city authority's employment registers.

Practice 2

1 In some cases alternative answers may be correct. Students must justify their answers.
   a Quota sampling, since it is not possible to form a sample frame of the leaves on the tree. The botanist could create categories by location, or tree size.
   b Stratified sampling divided into strata by age
   c Systematic sampling using the school register as a sampling frame.
   d Stratified sampling, taking 15 teachers, 2 administrative staff and 3 facilities staff.
   e Systematic sampling divided into strata by e.g. age/gender

2 Student's own list of advantages and disadvantages.
   They are not random samples, since in the first case everyone gets a feedback form, and in the second case only those that left their telephone number can be sampled.

3 a This sample could be very biased since height may affect athletic ability.
   b This could be suitable, depending on the gender distribution within the group.
   c Number the students from 1 to 150.
   Use a random number generator to generate 25 random numbers between 1 and 150.
   Choose the corresponding students.
   d Divide the year group into natural subgroups, such as male/female or by month of birth. Take random samples from these subgroups in the same proportion as they appear in the population.

Practice 3

1 a Chemistry mean = 66.2, Physics mean = 66.8
   b The tutor's claim is supported by the data but the difference between the two scores is very small. It would be better to say that the average scores in Physics and Chemistry are roughly the same.

c I have taken the scores in the last six tests to be representative of my ability.

2 a Quota sampling

b Pupils who are first to the canteen may not be representative of the whole population of pupils.

c Boys' mean spend: $3.73
Girls' mean spend: $3.14
This sample suggests the claim may be true.

d I have used the sample mean to infer things about the population mean.

Practice 4

1 a On average, the train is faster.

b The bus.

c He is more likely to be late using the train, due to the variability of the journey times.

2 a Programme A is faster.

b i Programme A. It never takes longer than 12.5 seconds.

ii Programme B. Programme A never takes less than 11.5 seconds but Programme B sometimes does.

iii Programme A. On average it is faster.

3 a Mean for Machine A = 1000.2, Mean for Machine B = 1005.3

b IQR for Machine A = 6.7, IQR for Machine B = 11.75

c On average, Machine A is more accurate and more consistent.

Mixed practice

1 Tom's marks are more consistent, but the IQR tells us nothing about who has done better. Tom may be consistently bad.

2 a On average, students in the chess club are taller, but they are also more variable.

b The IQR is not affected by extreme values.

c There is simply not enough information about the clubs to attempt to make these sorts of inferences; the ages and genders of the students within the clubs will have a large effect. Furthermore, while physical attributes are unlikely to have much effect on progression from youth chess clubs to adult chess playing, adult ballet dancers are selected on athletic and physical grounds.

3 a Quota sampling

b He will catch fish and measure them until he has 20 salmon and 30 trout.

4 e.g. The sample may be biased because the only people who will respond will be those who do not consider the survey 'unwanted'. People may have a similar attitude to surveys as they do to advertising. People who have strong feelings about unwanted advertising are likely to have asked to be removed from the directory.
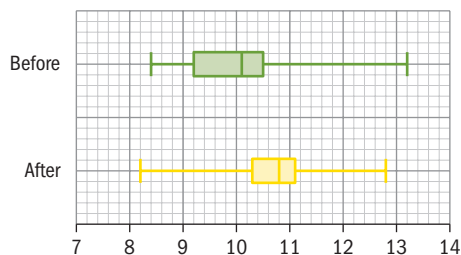
Review in context

1 a
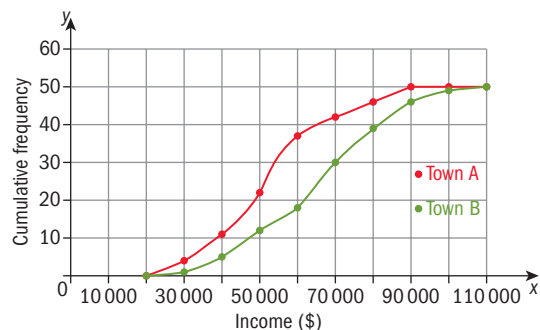
b On average, town B has higher household incomes. There is less variability though in town A.

c Different types of people were chosen for the sample in the same proportion as they appear in the population.

2 a

b Overall range similar; after data has smaller IQR so more consistent; after data has higher median.

c The intervention appears to have been effective.

d Small sample, limited age range covered.

3 a, b Student's own investigations.

c To take a census, you would use the data for every country in the world.

4 a The mean has increased from 5.6 to 6.4 hours, suggesting the scheme has been successful. Now, all residents report at least two hours of social contact, so everybody hits a minimum standard.

b Quota sampling.

c There are obvious flaws: those people using the bus already have a degree of mobility; just asking those who appear over 75 is not objective; asking if people would be interested is likely to bias the sample towards those who are likely to show improvement.

d e.g. To use a sampling frame from the electoral roll.

5 a e.g. People may not reliably remember what they donate to; people might be unwilling to share financial information.
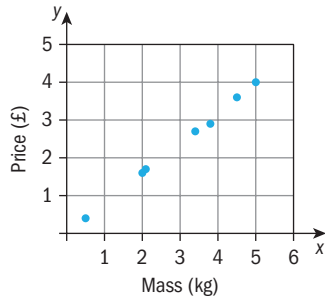
b Create a stratified sample according to where the parents grew up.

c 40 local, 60 from elsewhere and 33 from different countries will make a stratified sample of 133.

9.2

You should already know how to:

1 a

2 4.3

3 20: it is more than 1.5 times the IQR above the upper quartile
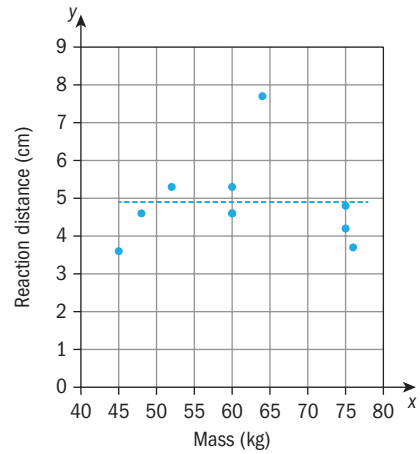
Practice 1

1 a No linear correlation – there is no correlation between scores in Mathematics and Art

b Weak positive correlation (linear) – there is a weak correlation between height and shoe size

c Strong positive correlation (linear) – there is a strong correlation between the amount of revision done and the score in the test

d Weak negative correlation (linear) – there is a weak negative correlation between the mass of the car and the gas mileage

e Strong negative correlation (linear) – the heating bill goes up as the temperature goes down.

f Moderate, non-linear association – the probability of having a short or tall height is low, however the probability of having a height in the middle is higher.

Practice 2

1 a b i
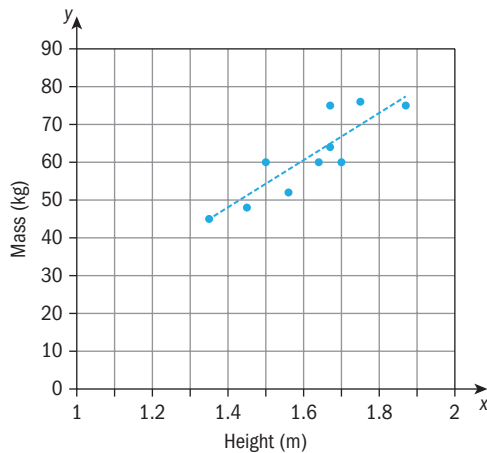
a b ii

a b iii
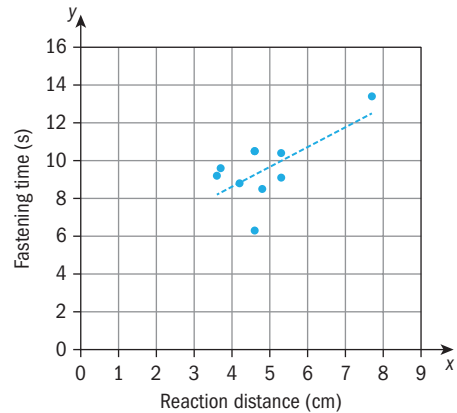
a b iv
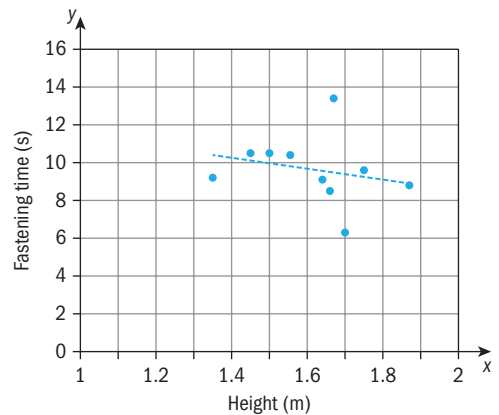
c i Strong positive linear correlation

ii No correlation

iii Weak positive correlation

iv Weak negative correlation

d 65 kg

e 7.1 seconds

f Graph **i** since it has the strongest correlation
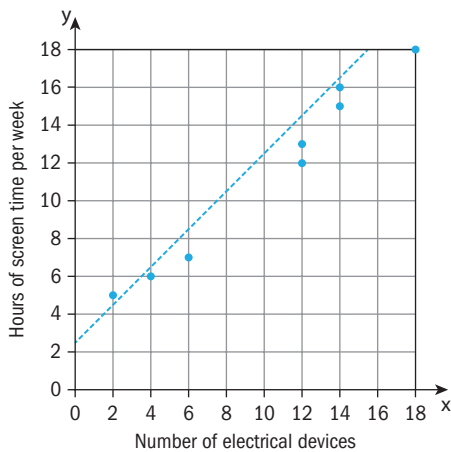
2 a b Weak positive correlation

Vehicles

c 20–22

Practice 3

1

Number of electrical devices

$\bar{x} = 10.25$
$\bar{y} = 11.5$

Number of electrical devices
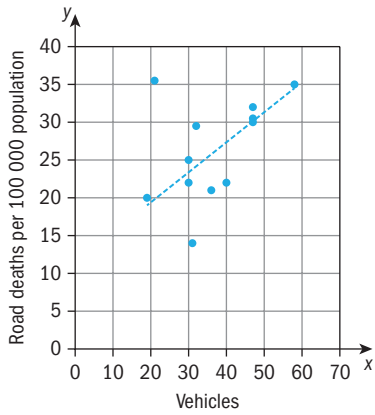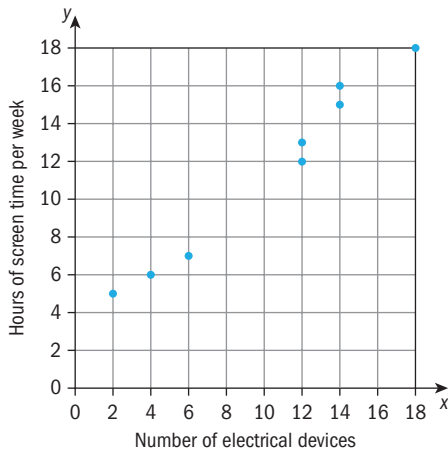
approximately $y = x + 2.5$
if $x = 10$, $y = 12.5$

2

Number of laps

$\bar{x} = 9.4$
$\bar{y} = 9.3$

Number of laps

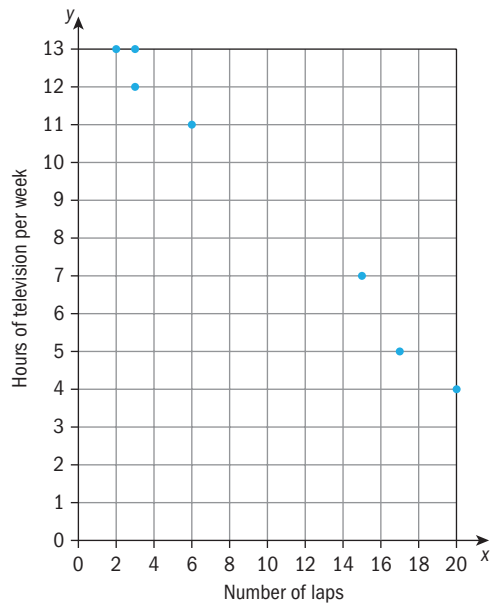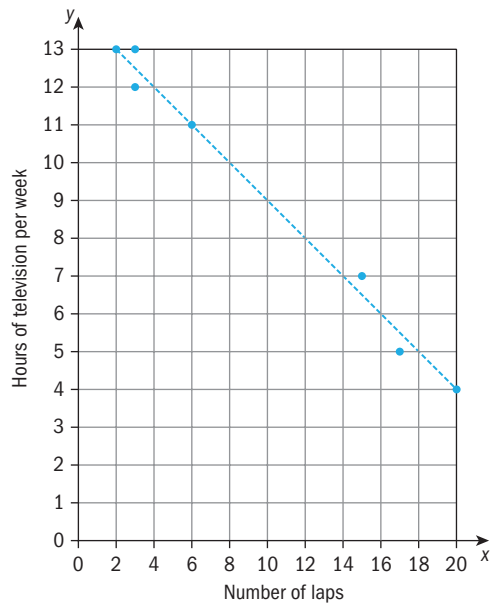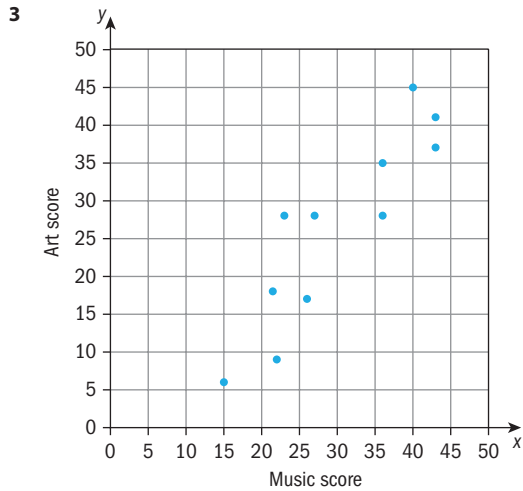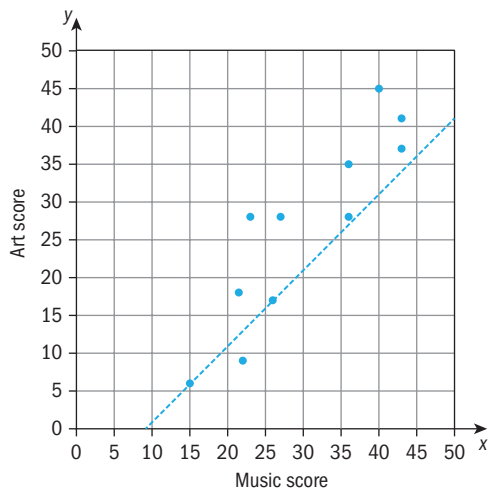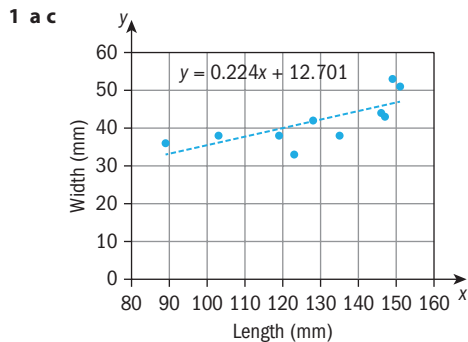approximately $y = -0.5x + 14$
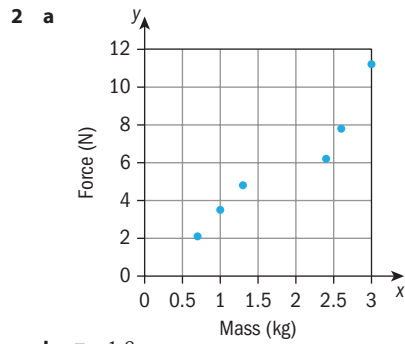if $x = 10$, $y = 9$

3

$\bar{x} = 30.3$
$\bar{y} = 26.5$

approximately $y = x - 9$
if $x = 10$, $y = 1$

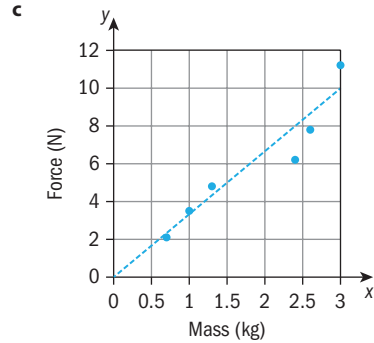It would not be valid as a music score of 10 is outside the range of music scores.

Mixed practice

1 a c

$y = 0.224x + 12.701$

b Moderate positive correlation

2 a

b $\bar{x} = 1.8$
$\bar{y} = 5.9$

c

d Predictions can be made as long as the predictions are in the range given.
e When $x = 1.5$, $y = 5$

3 a Positive, moderately strong, linear correlation.
b Yes, it goes through most of the points, similar amount above and below the line.
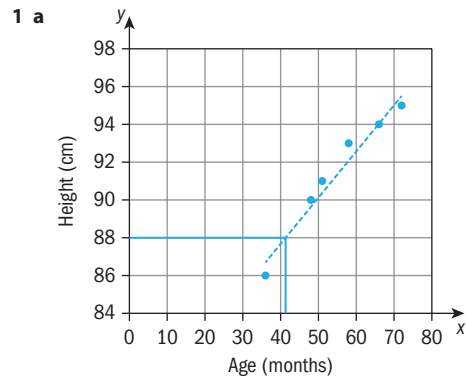**c** 610 kg
d 146 cm
e Approximately $y = 0.95x + 400$
f $y = 0.95x + 400$
$y = 0.95(138) + 400$
$= 531$ kg
g The line would only give valid predictions if the height of the horse was between 142 and 168 cm.

Review in context

1 a

b 88.2 cm
c No – outside the range of data – relationship may not continue
2 a Moderate negative correlation   **b** Approx 48 min
c No – outside the scope of the data – relationship unlikely to continue that far out.